Introduction to Problem Solving

I. Introduction
II. Types of Problems
III. Types of Solutions
IV. A Model of Problem Solving
V. Some Classic Problem Solving Terms

I. Introduction (cont)

A. Definition of a Problem
1) A set of given information:
   A description of the problem
2) A set of operations:
   Permissible actions or moves
3) A goal:
   A description of what constitutes a solution.

B. Problem Space (Ernst & Newell, 1969; Newell & Simon, 1972): a representation of the problem, often in the form of a tree diagram, with possible states and associated operations.

\[
3(x+1) = 2 \\
-2 \quad \div 3 \quad \div (x+1) \\
3(x+1)-2=0 \quad x +1=2/3 \quad 3=2/(x+1) \\
\]

I. Introduction (cont)
II. Types of Problems

A. Degree of Constraint

1) **Well Defined Problems**: The given information, the operations, and the goal state are completely specified.

Example: \( 3x = 2 \), solve for \( x \).

Well defined problems can be represented in a "problem space" (Newell & Simon)

A. Degree of Constraint (cont)

2) **Ill Defined Problems**: Problems in which there is uncertainty in either: the given information, the permissible operations, or the final state. Because of the uncertainty, the problem space cannot be completely specified.

Examples: cooking dinner

writing a term paper

etc.

II. Types of Problems (cont)

B. Classes of Problems (Greeno)

1) **Problems of inducing structure**:

\[ 1 4 2 4 3 4 4 4 5 4 6 4 7 \] (what is the next number)

\[ 8 5 4 9 1 7 6 3 7 0 \] (what is the missing number)

2) **Transformation Problems**:

- **water jar problems**:
  - Jars: A, B, C
  - size: 28, 7, 5
  - How do I obtain 11 units of water?
  - A-B-2C

3) **Arrangement Problems**:

- anagrams: eftca
- facet
II. Types of Problems (cont)

C. Solution Length: number of steps required to solve a problem

Example: A Farmer and his Animals

Imagine that you are a farmer with a goose, a fox, and some grain. You have to get across a river with all your belongings. However, you can only take one thing on the boat at a time.

Here is the problem. You cannot leave the goose with the grain, or he will eat it. You cannot not leave the fox with the goose, or the fox will eat the goose. How can you get all your belongings safely to the other side?

Start

Move Fox

Move Goose

Move Grain

Goal

Move Fox

Move Grain

Move Goose

Move Grain

Move Goose

Move Fox

Move Goose

Move Empty Boat

The problem space for the Farmer and His Animals problem.

Missionaries and Cannibals Problem

• In this problem there are three missionaries and three cannibals on one side of a river. They need to cross to the other side. The boat to carry them across can only take two at a time.

• You need to be careful as you move the missionaries and the cannibals. If at any time the cannibals on one shore outnumber the missionaries, the cannibals will eat the missionaries.

How can you get all the people safely across the river?
III. Types of Solutions

A. Algorithms: sets of operations that can be applied systematically and exhaustively to generate a solution.

Example: Calculate the mean for the following set of scores: 3, 4, 2, 5, 3, 6

algorithmic solution: Sum the scores, divide by n

NOTE: algorithms exist only for well defined problems!

(but not all well defined problems have known algorithmic solutions.)

III. Types of Solutions (cont)

B. Heuristics: strategies, or rules of thumb, that can be applied to a problem that often help generate a solution.

NOTE: Heuristics do not guarantee success!

Example:
In Black Jack, when do you take another card?
Heuristic: stay at 15 or more.
III. Types of Solutions (cont)

C. Algorithmic vs Heuristic solutions to the same problems

1) finding your car keys
2) choosing the next move in chess
3) choosing the fastest way to work/school

IV. A Model of Problem Solving

A. Based on Newell & Simon (1972)

Read Problem

Encode Problem into working memory

Search LTM for algorithms & heuristics

Means-ends analysis

Examine appropriate algorithms & heuristics

Means-ends analysis: Identifying differences between the current state and the goal state and selecting operations that reduce these differences.
IV. A Model of Problem Solving

B. Encoding the problem in working memory

1) representations of verbal problems

"The price of a notebook is four times the price of a pencil. One pencil costs 30 cents less than the notebook. What is the price of each?"

Algebraic Form: \( n = \text{notebook}, \ p = \text{pencil} \)

\( n = 4p, \ p = n - 30, \)

therefore \( p = 4p - 30 \)

\(-3p = -30 \)

\( p = 10, \ n = 40 \)

1) representations of verbal problems (cont)

Visual representation

2) Adding structure

The nine dot problem: connect the dots, with four straight lines, without lifting your pencil or pen.

```
+ + +
+ . +
+ . +
```
The nine dot problem (solution)

The problem is easier if the dots do not look like a square.

3. Syllogistic reasoning

<table>
<thead>
<tr>
<th>Problems</th>
<th>For each, choose an answer:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. All A are B</td>
<td>a) All A are C</td>
</tr>
<tr>
<td>Some B are C</td>
<td>b) Some A are C</td>
</tr>
<tr>
<td></td>
<td>c) Some A are not C</td>
</tr>
<tr>
<td>2. All A are B</td>
<td>d) None of the above</td>
</tr>
<tr>
<td>All C are B</td>
<td></td>
</tr>
<tr>
<td>3. Some B are A</td>
<td></td>
</tr>
<tr>
<td>Some C are not B</td>
<td></td>
</tr>
</tbody>
</table>
3. Syllogistic reasoning (cont)

Incorrect conversion of the premisses, probably due to the limited capacity of working memory:

\[\text{e.g. All A are B} \quad \text{or} \quad \text{A,B}\]

3. Syllogistic reasoning (cont)

Is this syllogism valid

All poisonous things are bitter.
Arsenic is not bitter.
Therefore, arsenic is not poisonous.

Failure to discriminate between the premises (in working memory) and relevant information in permanent memory.

V. Classic Terms

A. Functional Fixedness:

The candle problem (Dunker, 1945)

Task: fix a candle to the wall and light it

Materials: matches, candle, matchbox filled with thumbtacks.

Solution?

Result: people more likely to solve the problem is the matchbox is given empty, with the thumbtacks lose.
V. Classic Terms

B. Problem Solving set:
example Luchins ‘water jar’ problems

<table>
<thead>
<tr>
<th>Capacity of given jars</th>
<th>Amount to Get</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>1.</td>
<td>21</td>
</tr>
<tr>
<td>2.</td>
<td>18</td>
</tr>
<tr>
<td>3.</td>
<td>9</td>
</tr>
<tr>
<td>4.</td>
<td>20</td>
</tr>
<tr>
<td>5.</td>
<td>23</td>
</tr>
<tr>
<td>6.</td>
<td>15</td>
</tr>
<tr>
<td>7.</td>
<td>28</td>
</tr>
</tbody>
</table>

V. Classic Terms

B. Problem solving set: inappropriate application of past problem solutions to new problems.

V. Classic Terms

C. Incubation: the time between first beginning to work on a problem and returning to the problem provides new insights, or otherwise facilitates, the problem solving process.

- release from a problem solving set, or functional fixedness
- retrieval of new information by changing context
- recovery from fatigue
- conscious problems solving during absence
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