I. Background

Most typically in research we do not know either the mean or the standard deviation of the population. Both values must be estimated from our sample or samples from the population.

II. Two types of Experimental Designs

Population \( \mu_1, \sigma \)

Treatment

Population \( \mu_2, \sigma \)

We need estimates of \( \mu_1 \) and \( \mu_2 \), and \( \sigma \).

III. Difference Scores

IV. New formula for \( t \)

V. Hypothesis Testing

VI. Comparing the 2 sample and paired \( t \) tests

VII. Confidence Intervals
Between Subjects Design (two independent samples)

Population $\mu_1, \sigma_1$

Treatment

Population $\mu_2, \sigma_2$

Sample 1

Sample 2

Treatment 1 (no treatment control)

Treatment 2

Estimate $\mu_1$ from the sample $M_1$

Estimate $\mu_2$ from the sample $M_2$

Within-Subjects Design Related Samples

Population $\mu_1$

Treatment

Population $\mu_2$

Treatment

Sample 1 $X_{1.1}, X_{2.1}, \ldots$

Sample 1 $X_{1.2}, X_{2.2}, \ldots$

Estimate treatment effect $\mu_D$ from $M_D$, mean of the difference scores

Examples:

Within groups designs:
pretest-postest (diet, exercise)
repeated measures (taboo memory effect)

Matched groups designs:
yoking studies (executive monkeys)
Difference Scores

<table>
<thead>
<tr>
<th>Participant</th>
<th>First Score</th>
<th>Second Score</th>
<th>Difference Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X₁₁</td>
<td>X₁₂</td>
<td>X₁₂ - X₁₁ = D₁</td>
</tr>
<tr>
<td>2</td>
<td>X₂₁</td>
<td>X₂₂</td>
<td>X₂₂ - X₂₁ = D₂</td>
</tr>
<tr>
<td>3</td>
<td>X₃₁</td>
<td>X₃₂</td>
<td>X₃₂ - X₃₁ = D₃</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>n</td>
<td>Xₙ₁</td>
<td>Xₙ₂</td>
<td>Xₙ₂ - Xₙ₁ = Dₙ</td>
</tr>
</tbody>
</table>

Mᵣ = mean difference scores

Difference Scores

Mean difference: \( M_D = \frac{\sum D}{n} \)

\( SS_D = \sum (D - M_D)^2 \) (definition formula)

\( SS_D = \sum D^2 - \frac{(\sum D)^2}{N} \) (computational formula)

Variance of the differences scores:

\( s_D^2 = \frac{SS_D}{n-1} \)

Standard error of the difference scores:

\( s_{D/n} = \frac{s_D}{\sqrt{n}} \)

New formula for \( t \)

\( t = \frac{M_D - \mu_0}{s_{D/n}} \)

Note: \( \mu_0 \) usually equals 0
Hypothesis Testing

1. State H₀, H₁, and choose α
   - H₀: no treatment effect, μ₀ = 0
   - H₁: there is treatment effect,
     a) μ₁ ≠ 0
     b) μ₁ > 0
     c) μ₁ < 0

2. Determine what type of observation it would take to reject H₀
   a) What is the appropriate test statistic?
      Use the related samples t when you do not know the population parameters, and you have 2 closely related samples (e.g., within subjects design).
   b) What is the critical value?
      df = n - 1, n = number of pairs of observations, lookup t
Hypothesis Testing

3. Evaluate the sample data
   a) Calculate the differences scores D
   b) Calculate sample \( M_D \) and \( SS_D \)
   c) Calculate the \( S_D^2 \)
   d) Calculate the standard error \( S_{M_D} \)
   e) Calculate \( t \)

4. Reach a conclusion

Example:
Does hypnotherapy reduce the number of cigarettes smoked. Test at the .05 level.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Before Hypnotherapy</th>
<th>After Hypnotherapy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
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<tr>
<td>3</td>
<td>20</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>31</td>
<td>24</td>
</tr>
</tbody>
</table>

Comparing the 2-sample and paired \( t \) tests

Rework the same problem, but assume the data was from two independent groups of participants.
We have been testing hypotheses concerning population means.

\[ \mu_1 = \mu_2 \]

\[ \mu_1 - \mu_2 = 0 \]

\[ \mu_D = 0 \]

But what are: \( \mu_1 \); \( \mu_2 \); or \( \mu_0 \) ?

Our best estimates of the population values are obtained from our sample values (e.g., \( M_1 \) is our best guess of \( \mu_1 \)).

But these estimates are subject to error, and the size of that error is determined by sample size (law of large numbers).

The central limits theorem tells us that our estimate is determined by the standard error of the mean.

Confidence intervals give a range of scores around the sample mean within which we think the population value lies. High probability that the population mean is in this region.
Confidence Intervals

We compute this range by converting our scores to a known probability distribution, like $Z$ or $t$.

Example of 95% confidence interval with $Z$.

Scores that are unlikely to be obtained if the population mean is 400.

Confidence Intervals

Steps in constructing confidence intervals:
1. Determine interval size (e.g., 95%)
2. Look up appropriate statistic, using the two tailed value (e.g., a $t$ score).
3. Confidence interval is given by:
   $CI = M +/- t$(standard error)
Confidence Intervals

Example:
Construct the 95% confidence intervals for the smoking study presented earlier.

$t$ values for construction of 95% confidence interval with 3 degrees of freedom.

Middle 95% of $t$ distribution

$df = 3$
$p = .05$ in two-tails

$-3.182$ $+3.182$

Confidence Intervals

$CI = 5 +/- 3.182 (2)$
$CI = -1.364$ to $11.364$