Probability
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Probability & Statistics
Sampling from a population is a probabilistic process.

e.g., political polls

Probability & Statistics
Inferential Statistics are often stated as probabilities.

"The proportion of emotional words recalled (M = .35) exceeded the proportion on neutral words recalled (M = .27), F (1, 78) = 9.74, p < .01."
Definition

\[ P(A) = \frac{\text{The number of ways } A \text{ can occur}}{\text{The number of possible outcomes}} \]

Examples: coins, cards, dice

Contingency Tables

An enumeration of the possible outcomes of multiple events. e.g., blue jeans and gender.

<table>
<thead>
<tr>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td></td>
</tr>
</tbody>
</table>

Contingency Tables and Probabilities

Define:
- Probability of Jeans = \( P(A) \)
- Probability of Male = \( P(B) \)
- Probability of Male and Jeans = \( P(A \& B) \)
- Probability of Jeans given Male = \( P(A \mid B) \)
Key Terms

Independence:
A and B are independent if and only if
\[ P(A) = P(A\mid B) \]

Sampling with and without replacement:
Note, repeated samples are only independent if sampling is done with replacement.

Key Terms

Random Sample:
Every individual in the population has an equal chance of being sampled.

This requires sampling with replacement.

Rules of Probability

Counting rules (enumerating the outcomes).

Addition Rule
\[ P(\text{A or B}) = P(A) + P(B) - P(A\cap B) \]

Examples:
\[ P(2 \text{ coins, 2 heads or 2 tails}) = \]
\[ P(\text{heart or an ace}) = \]
Rules of Probability

Multiplication Rule

If A and B are independent:
\[ P(A \& B) = P(A) \times P(B) \]

Otherwise:
\[ P(A \& B) = P(A) \times P(B|A) \]

Examples (with independence):
- \[ P(\text{heart} \& \text{king in one draw}) = \]
- \[ P(2 \text{ heads in two tosses}) = \]
- \[ P(2 \text{ with two dice}) = \]

Examples (without independence):
- \[ P(\text{ace and ace, sampling w/o replacement}) = \]

Conjunction Fallacy

Tversky & Kahneman (1983)

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?
- Linda is a bank teller.
- Linda is a bank teller and is active in the feminist movement.
> \( P(A) \) = probability of a bank teller
> \( P(B) \) = probability of a feminist
> \( P(B|A) \) = probability of a feminist given she is a bank teller (must be less than 1)

> \( P(A \& B) = P(A) \times P(B|A) \)
> \( P(A \& B) \) cannot be greater than \( P(A) \)

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**Rules of Probability**

Addition and Multiplication rule work together:

Dice, \( P(\text{throwing 7's}) \):
Probability and the Normal Distribution

P(sampling a value as extreme as X from a distribution with known $\mu$ and $\sigma$)

Convert $x \rightarrow Z$ and $Z \rightarrow$ probability

Examples: $\mu = 80$, and $\sigma = 10$

$p(x > 85) =$

$p(x < 95) =$

$p(70 < x < 90) =$ (hint: use the addition rule)

The Binomial Distribution

When we are dealing with two exclusive outcomes (binary outcomes), the distribution of those outcomes forms the binominal.

Examples: heads/tails; male/female; correct/incorrect;

Focus: The number of heads in $n$ tosses of a coin.

<table>
<thead>
<tr>
<th>Coin</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>T</td>
<td>H</td>
</tr>
<tr>
<td>T</td>
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<td>T</td>
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<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of heads in 2 coin tosses</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>1</td>
<td>0.50</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Note the shape of the distribution as \( n \) gets large.

Normal Approximation to the Binomial

There are two outcomes (A, B), and \( n \) trials. Let \( p = P(A) \) and \( q = P(B) \). If \( np \) and \( nq \) are both greater than or equal to 10, then the binomial approaches the normal distribution with a mean equal to \( np \), and a standard deviation \( \sqrt{npq} \).

\[
Z = \frac{X - np}{\sqrt{npq}}
\]

\[
Z = \frac{X - \mu}{\sigma}
\]
Normal Approximation to the Binomial

Examples:
P(more than 30 heads in 50 tosses)

In a presidential poll of 20 people randomly sampled from the population, if there was no preference, what is the probability that 16 or more in the poll would favor candidate A?