I. Why Z

To compare 1 score to other scores in a distribution.

- How did I do on the GRE?
- Determine the effect of a treatment
  - Compared to “normal,” what impact did a drug have on my blood pressure?
- Compare scores measured on different scales.
  - Am I better at math or English?

II. The Standard Normal Distribution

- Lot’s of events conform to the normal distribution, and Z scores enable you to transform scores to this distribution.
- http://www.youtube.com/watch?v=6YDH6EVIvMs
II. The Standard Normal Distribution

III. Calculating $Z$

$Z = \frac{X - \mu}{\sigma}$

Creates a new distribution with a mean $= 0$ and a standard deviation equal to 1.
What does a Z measure

1. The distance of a score from the mean in standard deviation units.
2. We can use the score to compare people across different sets of scores.

Example:

<table>
<thead>
<tr>
<th>Class 1</th>
<th>Class 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>your grade</td>
<td>37/40</td>
</tr>
<tr>
<td>μ</td>
<td>28</td>
</tr>
<tr>
<td>σ</td>
<td>6</td>
</tr>
<tr>
<td>Z</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Application: Swimming

<table>
<thead>
<tr>
<th>100 Back</th>
<th>200 Back</th>
</tr>
</thead>
<tbody>
<tr>
<td>59.66</td>
<td>02:09.4</td>
</tr>
<tr>
<td>01:01.2</td>
<td>02:11.1</td>
</tr>
<tr>
<td>01:01.3</td>
<td>02:11.2</td>
</tr>
<tr>
<td>01:01.5</td>
<td>02:13.4</td>
</tr>
<tr>
<td>01:01.6</td>
<td>02:15.0</td>
</tr>
<tr>
<td>01:02.7</td>
<td>02:15.6</td>
</tr>
<tr>
<td>01:02.8</td>
<td>02:16.0</td>
</tr>
<tr>
<td>01:02.9</td>
<td>02:16.3</td>
</tr>
<tr>
<td>01:03.0</td>
<td>02:16.7</td>
</tr>
<tr>
<td>01:03.4</td>
<td>02:17.0</td>
</tr>
</tbody>
</table>

| 59.66 | 02:09.4 |
| 01:01.2 | 02:11.1 |
| 01:01.3 | 02:11.2 |
| 01:01.5 | 02:13.4 |
| 01:01.6 | 02:15.0 |
| 01:02.7 | 02:15.6 |
| 01:02.8 | 02:16.0 |
| 01:02.9 | 02:16.3 |
| 01:03.0 | 02:16.7 |
| 01:03.4 | 02:17.0 |
### IV. Calculating a score from Z

\[ Z = \frac{X - \mu}{\sigma} \]

- \( Z \sigma = X - \mu \)
- \( Z \sigma + \mu = X \)

#### Example: What would I have to score on a test, with mean = 28, and a standard deviation of 6, to have a Z = 2.0?

\[ X = Z \sigma + \mu \]

\[ X = 2 \times 6 + 28 \]

\[ X = 40 \]
V. Z and Standard Scores

Z are hard to work with
- positive and negative values
- no one wants to get a zero

To address this, scores are often converted to standard scores that have the properties desired.

\[ X_{\text{standard}} = Z \sigma_{\text{stand}} + \mu_{\text{stand}} \]

Example: SAT scores

Standardized to have a
\( \mu = 500 \) and \( \sigma = 100 \)

<table>
<thead>
<tr>
<th>Raw Score</th>
<th>Z score</th>
<th>Standard Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>( -1 )</td>
<td>( 530 )</td>
</tr>
<tr>
<td>70</td>
<td>( -0.5 )</td>
<td>( 550 )</td>
</tr>
</tbody>
</table>

Note, you’re not changing the shape of the distribution; you’re changing the mapping of numbers to scores.
Examples of Standard Scores

SAT ($\mu = 500; \text{ and } \sigma = 100$)
ACT ($\mu = 20; \text{ and } \sigma = 5$, approx)
IQ ($\mu = 100; \text{ and } \sigma = 15$)
Wechsler Adult Intelligence scale (Wais)
Wechsler Intelligence Scale for Children (Wisc)
Behavior Assessment System for Children (BASC) ($\mu = 50; \text{ and } \sigma = 10$)
Beck Youth Inventories ($\mu = 50; \text{ and } \sigma = 10$)
Wisc and Wais subscales ($\mu = 10; \text{ and } \sigma = 3$)

Summary

\[ Z = \frac{X - \mu}{\sigma} \]
\[ \mu = 0, \sigma = 1 \]
\[ X = Z \sigma + \mu \]

VI. Z and percentiles

The standard unit normal distribution
- Normal curve (bell shaped)
- Mean = 0
- Standard Deviation = 1
The area under the distribution can be easily calculated and is tabulated in normal distribution tables.
http://davidmlane.com/hyperstat/z_table.html
Table of Z (p. 699)

You can use this table to go from a Z score to a percentile, and from a percentile to a Z.
**Examples**

What is the Z score for:
- 30 percentile
- 50 percentile
- 75 percentile

What is the percentile rank for:
- Z = 1
- Z = -.5

If my IQ is 125, what is my percentile rank?

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**Application: Climate Change**

Extent of arctic sea ice by month (NOAA)

http://nsidc.org/arcticseaicenews/charctic-interactive-sea-ice-graph/

- 2012
- 2015
- Mean
  - (1981-2010)

Sept 19
Mean = 6.318 million square kilometers
Stdev = ~ .905
2015 = 4.507 (Z = -2.00)
2012 = 3.418 (Z = -3.20)

What percent have more coverage?
2015 = 2.28%  2012 = .07%