Decomposing complete graphs into cubes

Saad I. El-Zanati* and Charles Vanden Eynden
Illinois State University

Abstract

This talk concerns when the complete graph on $n$ vertices can be decomposed into $d$-dimensional cubes, where $d$ is odd and $n$ is even. (All other cases have been settled.) Necessary conditions are that $n \equiv 1 \pmod{d}$ and $n \equiv 0 \pmod{2^d}$. These are known to be sufficient for $d$ equal to 3 or 5, but for larger values of $d$ a decomposition has been proven to exist for only sparse sets of $n$. We prove that for each odd $d$ there is an infinite arithmetic progression of even integers $n$ for which a decomposition exists.