List-Coloring Certain Complete $n$-Partite Graphs

Linda Eroh
University of Wisconsin Oshkosh

Abstract

A graph is $k$-choosable if, for any assignment of lists of length $k$ to each vertex, there is a proper coloring of the graph in which each vertex is assigned a color from its list. A graph is equitably $k$-choosable if, for any assignment of lists of length $k$ to each vertex, there is a proper coloring of the graph in which every vertex is assigned a color from its list so that no color is used more than $\left\lceil \frac{n}{k} \right\rceil$ times, where $n$ is the order of the graph. We show that if every graph in some hereditary set of graphs is equitably $k$-choosable, then every graph in that set is equitably $k+1$-choosable. However, we conjecture that there is a graph, $K_{3,3,2,2}$, that is equitably 4-choosable but not equitably 5-choosable. This conjecture depends on whether $K_{3,3,2,2}$ is 4-choosable or not. We show that $K_{n(2)}$ is $n$-choosable and $K_{n(3)}$ is not $n$-choosable, for any positive integer $n$. Furthermore, $K_{1(3),n-1(2)}$ is $n$-choosable for any positive integer $n$. However, the status of $K_{3,3,2,2}$ is still open.

keywords: equitably $n$-choosable, $n$-choosable, list coloring, vertex coloring