The Steiner Problem in the hypercube

Tao Jiang *, Dan Pritikin
Miami University

Abstract

The Steiner problem seeks, for given a set $S$ of vertices in a connected host graph $G$, a tree of minimum size that contains all of $S$. Such a tree is called a Steiner tree for $S$. The Steiner problem has been extensively studied and has important applications in as diverse areas as VLSI-layout and phylogenetic trees.

Here, we consider the Steiner problem in the hypercube. Given a set $S$ of vertices in the $n$-dimensional cube $Q_n$, let $L(S)$ denote the size of a Steiner tree for $S$. Let $f(n, k)$ denote the maximum value of $L(S)$ over all sets $S$ of $k$ vertices. For all $k$ relatively small compared to $2^n$, we obtain asymptotically tight bounds on $f(n, k)$. Our upper bound is of an algorithmic nature while the lower bound is probabilistic. We will also briefly discuss the general behavior of the function $L(S)$. 

1