On H-linked graphs

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Abstract

We introduce the notion of $H$-linked graphs, where $H$ is a fixed multigraph with vertices $w_1, \ldots, w_m$. A graph $G$ is $H$-linked if for every choice of vertices $v_1, \ldots, v_m$ in $G$, there exists a subdivision of $H$ in $G$ such that $v_i$ is the branch vertex representing $w_i$ (for all $i$). This generalizes the notions of $k$-linked, $k$-connected, and $k$-ordered graphs.

Given $k$ and $n$, we determine the least integer $d$ such that, for every graph $H$ with $k$ edges, every $n$-vertex graph with minimum degree at least $d$ is $H$-linked. This value $D_1(k, n)$ appears to equal the least integer $d'$ such that every $n$-vertex graph with minimum degree at least $d'$ is $k$-connected. On the way to the proof, we extend a theorem by Kierstead et al on the least integer $d''$ such that every $n$-vertex graph with minimum degree at least $d''$ is $k$-ordered. We will also consider the connectivity conditions for a graph to be $k$-linked in the talk. This is a joint work with Prof. A. Kostochka.