1. The following polynomials are given, along with one root. Find the other roots by polynomial division and use of the quadratic formula.

a. \( P(x) = 2x^3 + 6x^2 - 2x - 6, \quad r_1 = 1 \)

b. \( P(x) = 2x^3 - 2x^2 - 2x + 2, \quad r_1 = -1 \)

c. \( P(x) = 2x^3 - 2x^2 + 2x - 2, \quad r_1 = 1 \)

2. Find a polynomial in standard form \( P(x) = ax^3 + bx^2 + cx + d \) that has the indicated roots:

a. \{-2,-1,3\}

b. \{-2i,2i,1\}

c. \{-2,-2,1\}

3. Find the a value for \( c \) such that -1 is a root of \( P(x) \)

a. \( P(x) = 2x^3 - 2x^2 + 3x + c, \quad r = -1 \)

b. \( P(x) = cx^3 - 2x^2 + 3x + 1, \quad r = -1 \)
4. The graph of polynomial $P$ is given in the adjacent figure. What can you say about the degree of $P$?

$\text{deg}(P)$ is odd/even*, and at least/most* ______.

5. Investigate the end behavior of the following polynomials:
   a. $P(x) = ax^5 - 2x^3 + 3x + 7$
      If $a > 0$ then $P(x) \rightarrow$ as $x \rightarrow \infty$
      and $P(x) \rightarrow$ as $x \rightarrow -\infty$
      If $a < 0$ then $P(x) \rightarrow$ as $x \rightarrow \infty$
      and $P(x) \rightarrow$ as $x \rightarrow -\infty$
      
   b. $P(x) = ax^8 - x^5 + 3x^2 - 3$
      If $a > 0$ then $P(x) \rightarrow$ as $x \rightarrow \infty$
      and $P(x) \rightarrow$ as $x \rightarrow -\infty$
      If $a < 0$ then $P(x) \rightarrow$ as $x \rightarrow \infty$
      and $P(x) \rightarrow$ as $x \rightarrow -\infty$

6. Find all roots, real and complex, of the polynomial
   a. $f(x) = (x - 1)^2(x + 2)(x^2 + 9)$
      Roots: ________________________________.
   b. $g(x) = (x^2 - 1)(x^2 + 2)(x - 3)^2$
      Roots: ________________________________.
7. Find the quotient and remainder by long division

a. \[ \frac{2x^4 - 3x^2 + 5x - 2}{x - 1} = \]

b. \[ \frac{2x^3 - x^2 + 2x - 1}{x^2 + 1} = \]
6. \( P(x) \) is a third degree polynomial:
\[
P(x) = a(x - r_1)(x - r_2)(x - r_3)
\]
with lead coefficient \( a = -\frac{1}{2} \) and roots \( r_1 = -2 \) and \( r_2 = 2 \).
If, additionally, it is known that the graph of \( P \) passes through the point \((1,3)\),
what is the value of the remaining root \( r_3 \)?
(Note: leave the expression for \( P \) in factored form. Expanding (‘foiling’) is not helpful here.)

\[ r_3 = \quad \]

7. Find a third degree polynomial \( P(x) \):
\[
P(x) = a(x - r_1)(x - r_2)(x - r_3)
\]
with roots \( r_1 = -1, r_2 = 1 \) and \( r_3 = 3 \) whose graph passes through the point \((2, 1)\).
You may leave the expression for \( P \) in factored form.

\[
P(x) = \quad \]

b. Compute the y-intercept of \( P(x) \)

\[ y = \quad \]
8. Consider the following data, representing the change in value of the Dow Jones Industrial Index (\(\text{DJIE}\)) on 9 consecutive working days \((x = 0 - 8)\):

<table>
<thead>
<tr>
<th>Day ((x))</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \text{DJIE})</td>
<td>3</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>-2</td>
<td>-4</td>
</tr>
</tbody>
</table>

We wish to model the data using a third degree polynomial \(P(x)\):

\[
P(x) = a(x - r_1)(x - r_2)(x - r_3)
\]

with roots \(r_1\), \(r_2\) and \(r_3\), whose graph approximates the data points.

a. Sketch the data, smoothly connect and estimate the roots of \(P(x)\) from your curve.

\[
r_1 = \quad ,
\]

\[
r_2 = \quad \text{and}
\]

\[
r_3 = \quad .
\]

b. If \(P(4) = 2\), what is the value of \(a\)?

\[
a = \quad .
\]

c. Plot the polynomial in the figure above. How does it compare to your graph?