1. Solve the system \( \begin{cases} ax + by = d \\ cx + ay = e \end{cases} \) for \( x \) and \( y \) using Cramer’s Rule.

2. For which values of \( c \) is the matrix \( \begin{bmatrix} c & 0 & c \\ 1 & c & 1 \\ c & 1 & 2 \end{bmatrix} \) singular?

3. The matrix \( A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \) has determinant \( |A| = 3 \).

Based on this information, find the values of the following determinants:

\[
\begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g - a & h - b & i - c \end{vmatrix} = \ldots \]
\[
\begin{vmatrix} 2a & 2b & 2c \\ 2(d + a) & 2(e + b) & 2(f + c) \\ 2(g - 3d) & 2(h - 3e) & 2(i - 3f) \end{vmatrix} = \ldots \]

\[
|A^{-1}| = \ldots \]
\[
|A^3| = \ldots \]
\[
|A + A| = \ldots \]

\[
\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = \ldots \]
\[
\begin{vmatrix} d & a & g \\ e & b & h \\ f & c & i \end{vmatrix} = \ldots \]
\[
\begin{vmatrix} a & d & d - 2a \\ b & e & e - 2b \\ c & f & f - 2c \end{vmatrix} = \ldots \]
4. Consider the matrix the matrix \( A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \).

a. Find the characteristic polynomial of \( A \).

b. Use the characteristic polynomial of \( A \) to show that 2 is an eigenvalue of \( A \).

c. Determine the eigenvector associated with the eigenvalue 2.

5. Prove that a matrix \( A \) is invertible if and only if 0 is not an eigenvalue of \( A \).
6. 100 rats are released in compartment A of a maze (see figure) at time \( t = 0 \).

The rats have been trained to randomly move to an adjacent compartment at the signal of a bell which rings every 15 minutes.

a. Determine the transition matrix \( A \) for the distribution of the rats.

b. Determine the distribution vector \( x \) for the rats, rounded to the nearest integer at time \( t \):

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
<th>150</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>[ ]</td>
<td>[ ]</td>
<td>[ ]</td>
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</tbody>
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C. Based on the steady state distribution of the rats (at \( t = \infty \)), what eigen information can you give for the transition matrix \( A \) ?

d. If 1000 rats are released in compartment A of the maze at time \( t = 0 \), what would be the steady state distribution of the rats?

e. If 100 rats are released in compartment D of the maze at time \( t = 0 \), what would be the steady state distribution of the rats?
7. Four vaults, A, B, C and D are connected by hallways (see figure). A security guard patrols the vaults as follows: At time \( t = 0 \) the guard is in vault A. Every 30 minutes, the guard randomly chooses a hallway and moves to the next vault.

a. Determine the probability matrix \( A \) for the location of the guard.

b. Determine the probability vector \( x \) for the position of the guard at time \( t \):

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
<th>300</th>
<th>( \infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>[ ]</td>
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</tr>
</tbody>
</table>

Use your calculator to compute \( |A^T + \mathbf{T}| = \) 

The result can be expressed as a well known advertising slogan in the cell phone industry: “\( A^T + \mathbf{T} \) is now ___________________”. Sorry.
9. The following information is given about the square matrix \( A \):
\[
\begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix}
\]
and 
\[
\begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}
\]
\( A \) are 
\( A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \) and 
\( A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2} \).

a. What are the eigenvalues \( \lambda_{1,2,3} \) and eigenvectors \( \{ p_1, p_2, p_3 \} \) of \( A \)?

\( \lambda_1 = \quad , \ p_1 = \quad ; \lambda_2 = \quad , \ p_2 = \quad ; \lambda_3 = \quad , \ p_3 = \quad ; \)

b. If a matrix has real, distinct eigenvalues, what can be said of its eigenvectors?

------------------------------------------------------------------------------------------------------------------.

c. Are the eigenvalues of \( A \) distinct? _______.  

Are the eigenvectors independent? _______. Show this.

Is this result in conflict with your statement under (b)? _______. Motivate your answer!

d. Compute the matrix \( A \) from the eigen information.

e. Find a basis for \( E_1 \), the eigen space associated with \( \lambda_1 \)

f. Find a basis for \( E_3 \), the eigen space associated with \( \lambda_3 \)

g. Without a calculator, use the eigeninfo to compute the entries of \( A^{10} \)