1. A semi circular city with radius 5 miles borders the ocean. (See figure).

a. Compute the average value of the distance of a point \( P(x,y) \) in the city to the ocean.

b. If the population density decreases linearly with distance from the ocean, from 10 (thousand per square mile) on the ocean front to 0 at 5 miles away, what is the total population size of the city?

c. What is the average population size (or density) of the city?
2. A sanctuary with a rectangular floor and a planar roof is constructed according to the dimensions in the figure:

   a. Compute the volume of the sanctuary as a triple integral

   b. If the temperature rises linearly from 20 F on the floor level to 28 F at the building's highest point, compute the average value of the temperature in the building.
3. A semi-circular city with radius 5 miles borders the ocean. The center of the region is a semi-circular bay with radius 1 mile. (See figure).

a. Suppose the price of housing at location $P$ in the city depends linearly on $d_{oc}$, the distance of $P$ to the ocean (not the bay):
   - at 5 miles housing costs $100 /sqft, while
   - ocean front property goes for $2000 /sqft. Compute the average price of housing in the city.
   (Hint: Sketch price vs $d$ to find the equation of the line. The solution benefits from the use of polar coordinates)

b. Suppose the price of housing at location $P$ in the city depends linearly on $d_{bay}$, the distance of $P$ to the bay (not the ocean):
   - at 4 miles from the bay, housing costs $100 /sqft while
   - bay front property goes for $3000 /sqft. Compute the average price of housing in the city.

c. Suppose the price of housing at location $P$ in the city depends linearly on both $d_{bay}$, the distance of $P$ to the bay ($d_{bay}$) and on the distance of $P$ to the ocean ($d_{oc}$) as:
   $p = 2000 + 250d_{bay} + 180d_{oc}$.
   Set up the integral for computing the average price of housing in the city in polar coordinates.

Compute the average price of housing in the city.
4. Sketch the region of integration, then compute the integral:

\[
\begin{align*}
a. \quad & \int_{y=0}^{y=4} \int_{x=0}^{x=y^2} x^2 y^3 \, dx \, dy = \\
& \int_{y=1}^{y=4} \int_{x=0}^{x=\sqrt{y}} x^2 \, dx \, dy = \\
& \int_{y=0}^{y=2} \int_{x=0}^{x=y^2} 2xy \, dy \, dx = \\
& \int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{x}} 2xy \, dy \, dx = \\
& \end{align*}
\]
5. Sketch the region of integration. Then compute the integrals by reversing the order of integration:

- **a.** \( \int_{x=0}^{1} \int_{y=-x}^{1} e^{x^2} \, dx \, dy \)

- **b.** \( \int_{x=0}^{\sqrt{3}} \int_{y=-x^2}^{x^3} y \sin(x^2) \, dx \, dy \)

- **c.** \( \int_{x=0}^{1} \int_{y=-x}^{x e} \frac{x}{\ln x} \, dx \, dy \)
6. Sketch the region of integration. Then convert the problem to polar coordinates and evaluate:

a. \[ x > 0, \quad y \leq \sqrt{y^2 - x^2} \]
   \[ \int_{x=0}^{x} \int_{y=0}^{\sqrt{y^2 - x^2}} x \, dy \, dx \]

b. \[ y > 0, \quad x \leq \sqrt{y^2 - x^2} \]
   \[ \int_{y=0}^{y} \int_{x=0}^{\sqrt{y^2 - x^2}} xy \, dx \, dy \]

c. \[ y > 0, \quad x \leq \sqrt{\frac{y^2}{x}} \]
   \[ \int_{x=0}^{x} \int_{y=0}^{\sqrt{y^2 - x^2}} y \, dy \, dx \]