Sample Questions Test 3

1. The problem \( \frac{d^2 y}{dx^2} + p(x) \frac{dy}{dx} + q(x)y(x) = f(x), \)
   has fundamental set \{y_1(x), y_2(x)\}.
   
   a. State the complementary solution of the problem.

   b. Give a particular solution of the problem.

   c. State the general solution of the problem.

2. The DE \( a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy(x) = 0 \) has solution \( y(x) = C_1 e^{\frac{x}{2}} \cos 2x + C_2 e^{\frac{x}{2}} \sin 2x. \)
   Determine values for the constants \( a, b \) and \( c. \)

3. Show whether the functions \{\( y_1(x), y_2(x) \)\} are linearly independent on the indicated interval.
   Clearly state your conclusion.
   
   a. \( \{y_1 = \frac{1}{\sqrt{x}}, \ y_2 = \frac{1}{\sqrt{2x}}\} \) on \((0, \infty)\)

   b. \( \{y_1 = e^{ax}, \ y_2 = e^{bx}\}, \ a \neq b \) on \((-\infty, \infty)\)
4. Find the solution of the initial value problem:
\[
\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 5y(x) = 0, \quad y(0) = 1 \quad \text{and} \quad y'(0) = -1
\]

b. Plot the graph of the solution on the interval [0,5].

5. Find the solution of the initial value problem:
\[
x^2 \frac{d^2y}{dx^2} + 2xy(x) + 2y(x) = 0, \quad y(1) = 0 \quad \text{and} \quad y'(1) = 1
\]

b. And plot the graph of the solution on the interval [0,5].
6. Consider the problem: \( \frac{d^2y}{dx^2} + p(x) \frac{dy}{dx} + q(x)y(x) = \sqrt{x}, \quad x > 0 \)

Given that the functions \( \{y_1 = \sqrt{x} \cos x, \quad y_2 = \sqrt{x} \sin x\} \)
form a fundamental set of solutions of the DE,

a. State the complementary solution of the problem

b. Find a particular solution of the problem by variation of parameters

c. State the general solution of the problem

d. Determine the value of any unknown constants
   from the initial conditions \( y(\pi/2) = 0 \) and \( y'(\pi/2) = 1 \)

e. Use your grapher to plot the solution
   on the interval \([0,5]\).
7. Consider the problem: 

\[ y''(x) + 2y'(x) + y(x) = \frac{e^x}{x} \]

a. Find a fundamental set of solutions of the DE and state the complementary solution.

b. Find a particular solution of the problem by variation of parameters.

c. State the general solution of the problem.

d. Determine the value of any unknown constants from the conditions \( y(0) = 1 \) and \( y'(0) = -1 \).

e. Use your grapher to plot the solution on the interval \([0,5]\).
8. Find the general solution of the initial value problems, where $\lambda$ is a positive constant:

$$\frac{d^2y}{dx^2} - \lambda^2 y(x) = e^{-\lambda x}, \quad y(0) = 1 \text{ and } y'(0) = -1$$

a. Find a fundamental set of solutions of the DE and state the complementary solution.

b. Find a particular solution of the problem by variation of parameters.

c. State the general solution of the problem.

d. Determine the value of any unknown constants from the initial conditions.

e. Use your grapher to plot the solution on the interval $[0, 5]$. 
9. Find the general solution of the initial value problems, where \( \lambda \) is a positive constant:

\[
\frac{d^2y}{dx^2} + \lambda^2 y(x) = \sin \lambda x, \quad y(0) = 0 \quad \text{and} \quad y'(0) = 1
\]

a. Find a fundamental set of solutions of the DE and state the complementary solution.

b. Find a particular solution of the problem by variation of parameters

c. State the general solution of the problem

d. Determine the value of any unknown constants from the initial conditions

e. Use your grapher to plot the solution on the interval \([0, 5]\).
10. The problem: \( \frac{d^2y}{dx^2} - \frac{1}{x} \frac{dy}{dx} + q(x)y(x) = 0, \quad x > 0 \) has solution \( y_1(x) = \sqrt{x} \).

a. Compute the unknown coefficient function \( q(x) \).

b. Find a second linearly independent solution \( y_2(x) \) by reduction of order

c. State the general solution of the problem

d. Determine the value of any unknown constants from the conditions \( y(1) = 1 \) and \( y'(1) = -1 \)