1. Show that \( \frac{2n}{n+1} \) converges to 2. Explicitly state the expression for \( N \) in terms of \( e \).

2. Show that \( \sqrt{n^2 + n} - n \) converges to the value \( \frac{1}{2} \).

3. a. Show that every convergent sequence is bounded.
   b. Give an example of a bounded sequence that does not converge.

4. Prove the Bounded Monotone Convergence Theorem (BMCT)

5. Flannery O’Connor wrote the book titled “Everything that rises must converge”
   Correct the title to reflect the BMCT.

6. a. Use a recurrence to construct a sequence for the iterated fraction
   \[
   \frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \ldots}}}.
   \]
   Use your calculator to find the value of the fraction.
   b. Use the Bounded Monotone Convergence Theorem to establish convergence of the fraction using a suitable starting value.

7. Show that if \( \lim a_n = a \) and \( \lim b_n = b \) and \( c \) is a real constant
   a. \( \lim ca_n = ca \)
   b. \( \lim(a_n + b_n) = a + b \)
   c. \( \lim(a_n b_n) = ab \)

8. Show that if \( \lim a_n = a \) and \( a_n > c \) for some constant \( c \) then \( a > c \)

9. a. (SQUEEZE THEOREM) Show that if \( a_n \leq c_n \leq b_n \) and \( \lim a_n = L \) and \( \lim b_n = L \)
   then \( \lim c_n = L \)
   b. Use the Squeeze Thm and the fact that \( -1 \leq \sin n \leq 1 \)
   to show that \( \left( \frac{\sin n}{n} \right) \) converges to 0

10. Show that the value of a limit, if it exists, is unique:
    That is, if \( \lim a_n = L \) and \( \lim a_n = M \) then \( L = M \)
11. Let \((a_n)\) be a bdd. (but not necessarily convergent) sequence.
   a. Let \((b_n)\) converge to 0. Show that \((a_n, b_n)\) converges to 0
   b. Let \((b_n)\) converge to \(L \neq 0\). Show that \((a_n, b_n)\) does not necessarily converge.

12. Use the Cauchy Condensation Test to prove that \(\sum \frac{1}{n^p}\) converges iff \(p > 1\).

13. Use Newton’s Method to construct a recurrence to estimate the value of \(\sqrt{2}\).
   Using the BMCT, show that this sequence converges.

14. a. Prove that if \((a_n) \to L\) then every subsequence \((b_n)\) of \((a_n)\), converges to \(L\).
   b. State the contrapositive of part (a).
   c. Prove that the sequence \((-1)^{n+1} \frac{n}{n+1}\) diverges by identifying two subsequences
      that converge to different values.

15. State and prove the BOLZANO-WIEIERSTRASS THEOREM, using the Nested Interval Property.

16. a. Show that if a sequence converges, then it is Cauchy.
   b. Show that a Cauchy sequence is bdd.
   c. Prove the converse of part (a).

17. Consider the alternating series: \(\sum (-1)^{n+1} a_n\) where \(a_n > 0\).
   State the ALTERNATING SERIES THEOREM (AST)
   Prove the AST by showing that the sequence of partial sums is Cauchy.

18. (RATIO TEST) If \(\lim \frac{a_{n+1}}{a_n} = r < 1\) then the series \(\sum a_n\) converges absolutely.

19. Show that, if \(\sum a_n\) converges absolutely and \((b_n)\) is a bdd sequence, then \(\sum a_n b_n\) converges.

20. Let \((s_n)\) be the sequence of partial sums of the harmonic series \(\sum \frac{1}{n}\)
   Consider the sequence \((t_n) = (s_n - \ln n)\)
   a. Show that \(\ln n\) represents the area under the curve \(f(x) = \frac{1}{x}\) from \(x = 1\) to \(x = n\)
   b. Interpret \(t_n\) as an area and graphically show that \(t_n > 0\)
   c. Interpret \(t_{n+1} - t_n\) as an area and graphically show that \(t_{n+1} - t_n < 0\)
   d. Use the Bdd Monotone Sequence Thm to establish convergence of \((t_n)\)
   e. The limit of \((t_n)\) is Euler’s Constant, \(\gamma = \lim(s_n - \ln n)\)
      What is the most your calculator will do for finding the value of \(\gamma\)?