**DEFINITION:** A set is a collection of objects, called elements.

The use of a set is indicated by the occurrence of curly brackets to signify that the order of the objects is of no importance. Generally, we use a capital letter to refer to a set:

\[ A = \{\text{Honda, Suzuki, Kawasaki, Yamaha}\} = \{h, s, k, y\} \]

In listing the elements of a set:
- the order in which the elements are listed is unimportant: \( \{h, s, k, y\} = \{k, s, h, y\} \)
- elements are listed only once, multiples are omitted: \( \{h, s, s, k, y, k, h\} = \{h, s, k, y\} \).

To denote membership of a set, we use the symbol ‘?’.
Thus, \( y \in A \) means: \( y \) is an element of the set \( A \) (or, for short, \( y \) is in \( A \)).

Sets can also be given in descriptor notation. The statement
\[ A = \{x \mid x \text{ is a Japanese motorcycle brand}\} \]

is read as:
\[ A : \text{“}A\text{”} \]
\[ = : \text{“}is\text{”} \]
\[ \{ : \text{“} the set\text{”} \]
\[ x : \text{“of all elements } x\text{”} \]
\[ \mid : \text{“for which”} \]
\[ \ldots \text{ description follows} \ldots \]

Cardinality refers to the number of elements in a set (or the ‘size’ of a set). It is denoted by vertical bars:
\[ |A| = 4 \text{ means: the set } A \text{ has 4 elements.} \]

Sets are seen as part (subsets) of the universal set \( U \), the set of all objects under consideration. The symbol for ‘subset’ is chosen to reflect the less than or equal sign for numbers:

\( A \subseteq U \) means \( A \) is a subset of \( U \). This implies that if an element \( x \) is in \( A \), then it must be in \( U \):
Thus, \( A \subseteq U \) means: \( x \in A \Rightarrow x \in U \)

Sets are often depicted with the help of Venn diagrams in which subsets are embedded in the larger set which contains them. Thus the statement \( A \subseteq U \) is illustrated by:

![Venn Diagram](image)

The complement of a set \( A \) is the set of all elements that are not in \( A \):
\( \overline{A} = \{x \mid x \notin A\} \), depicted by a Venn Diagram as:

![Venn Diagram](image)
Union and Intersection of sets.

**Definition:** The union of two sets A and B, denoted by \( A \cup B \), is the set of all elements that are in A or in B (or in both). Symbolically, \( A \cup B = \{ x \mid x \in A \text{ or } x \in B \} \)

Shade the union of A and B in the figure:

![Venn diagram for union]

**Definition:** The intersection of two sets A and B, denoted by \( A \cap B \), is the set of all elements that are in A and in B. Symbolically, \( A \cap B = \{ x \mid x \in A \text{ and } x \in B \} \)

Shade the intersection of A and B in the figure:

![Venn diagram for intersection]

**De Morgan’s Laws.**

To see whether complement distributes over union, draw Venn diagrams for the following sets:

\[
\overline{A \cup B} \\
\overline{A} \cup \overline{B}
\]

Comparison of the two illustrations immediately shows that:

To see whether complement distributes over intersection, draw Venn diagrams for the following sets:

\[
\overline{A \cap B} \\
\overline{A} \cap \overline{B}
\]

Comparison of the two illustrations immediately shows that:

Vertical comparison of the figures, however, shows the important identities:

**De Morgan’s Laws:** \( \overline{A \cup B} = \overline{A} \cap \overline{B} \) and \( \overline{A \cap B} = \overline{A} \cup \overline{B} \).
1. \( U = \{ \text{Red, Orange, Yellow, Green, Blue, Indigo, Violet} \} = \{ R, O, Y, G, B, I, V \} \)
   
   \( A = \{ \text{Orange, Yellow, Indigo} \} = \{ O, Y, I \} \)
   
   \( B = \{ \text{Red, Orange, Yellow, Green} \} = \{ R, O, Y, G \} \)
   
   a. \( \overline{A} = \)
   b. \( \overline{B} = \)
   c. \( A \cap B = \)
   d. \( A \cup B = \)
   e. \( A \setminus B = \)
   f. \( A \cap B = \)
   g. \( A \cup B = \)
   h. \( A \setminus B = \)
   i. Complete De Morgan's Laws (in terms of sets \( A \) and \( B \)): 
      \( A \cap B = \) and \( A \setminus B = \)

2. Using Venn-diagrams, show whether intersection distributes over union:
   \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \)
   
   by shading the appropriate region for the left hand side (LHS) in rectangle 1
   and for the right hand side in rectangle 2. Use a legend!

   ![LHS and RHS Venn Diagrams]

   Conclusion: ____________________________________________________________________ .

3. A survey of records from a pediatric clinic yielded the following results:
   
   25 children had been given an injectable polio vaccine
   57 children had been given an oral polio vaccine
   4 children had been given oral and injectable vaccines
   12 children had had no polio vaccine.

   Draw a Venn Diagram for the problem in the adjacent figure

   How many children participated in the survey? __________.
4. A music questionnaire had 50 respondents. Of these, 25 people indicated they like Country. 34 people indicated they like Rock while 5 said they do not like either one of them.

a. Draw a Venn Diagram for the problem in the above figure.

b. Of the ones interviewed, how many people like Country and Rock? ______________.

5. A form completed by members of an insurance plan contained the following statement:

I exercise by: Q Running  Q Swimming  Q Cycling  Q None of these

‘Running’ was checked 28 times
‘Swimming’ was checked 35 times
‘Cycling’ was checked 15 times
‘Running’ and ‘Swimming’ were checked 13 times on the same form
‘Running’ and ‘Cycling’ were checked 11 times on the same form
‘Swimming’ ‘Cycling’ were checked 7 times on the same form
‘Running’ ‘Swimming’ and ‘Cycling’ were checked 4 times on the same form
14 people checked ‘None of these’.

Draw a Venn diagram of the survey data:

a. How many people filled out the form? ______________.
b. How many people cycle and swim, and do not run? ______________.
c. How many people run or cycle, and do not swim? ______________.
d. How many people use cycling as their only exercise? ______________.
e. How many people use only one form of exercise? ______________.
6. The respondents of an insurance survey were classified according to their cardiac risk. The percentages at risk for the factors diet, stress and family history were:

- 63% were at risk because of diet
- 56% were at risk because of stress
- 23% were at risk because of family history
- 38% were at risk because of diet and stress
- 11% were at risk because of diet and family history
- 17% were at risk because of stress and family history
- 8% were at risk because of diet and stress and family history

Draw a Venn diagram of the survey data and assign the appropriate number to each region:

What percentage of those surveyed were at risk because of:

- a) only one risk factor? ...........................................
- b) diet and family history, but not stress? ..................
- c) diet and stress? .............................................
- d) diet or stress? .............................................
- e) What percentage of those surveyed were not at any of the 3 risks? ..........................
7. The respondents of an insurance questionnaire were classified according to their cardiac risk: The percentages at risk for the factors diet, stress and family history:

- 2% were at risk because of stress only
- 19% were at risk because of diet and stress but had no family history
- 10% were at risk because of stress and family history but not because of diet
- 7% were at risk because of stress and family history and diet
- 11% were at risk because family history and diet
- 43% were at risk because of diet
- 22% were at risk because of family history

a) Draw a Venn diagram of the survey data:

What percentage of those surveyed were at risk because of

a) only one risk factor? ____________________.
b) diet and family history, but not stress? ____________________.
c) diet and stress? ____________________.
d) diet or stress? ____________________.
e) What percentage of those surveyed were not at any of the 3 risks? ____________________.
8. A survey on the MTSU Knoll had 70 responses. Of these,
35 indicated they smoked
40 indicated they drank alcoholic beverages
15 indicated they neither smoked nor drank alcohol

a. Draw a Venn diagram of the survey data:

b. Of the ones interviewed, how many people indicated they
   Smoked and drank? _____________.
   Smoked and did not drink? _____________.

9. Consider the following subsets of the
   American population (U):
   
   A: people from an agricultural background.
   B: people who own a business
   C: people with two or more years of college

   Shade the following areas in the adjacent figure
   and describe the members of each group:

   $A \cap B$?
   $C$:
   __________________________________________________________________________.

   $A \cap B$?
   $C$:
   __________________________________________________________________________.

   $A$?
   $B$?
   $C$:
   __________________________________________________________________________.

   $A$?
   $B$?
   $C$:
   __________________________________________________________________________.

   $A$?
   $B$?
   $C$:
   __________________________________________________________________________.
10. A focus survey among County high school students used the following categories:

A: students with an academic focus.
S: students with sports focus
M: students with a music and arts focus

Represent the shaded areas in the adjacent figure symbolically and describe the members of each group:
(shading done in class ...)

11. Of 500 people registered for the South Eastern Region Mathematics Conference held at MTSU on November 2-3 at MTSU,

200 are members of the American Mathematical Society (AMS)
320 are members of the Mathematical Association of America (MAA)
while 60 people hold no membership in either the AMS or the MAA.

Assign the cardinalities in the following Venn diagram:

How many of those registered have a dual membership of both AMS and MAA?

| AMS ∩ MAA | ? |