**Program STEFFENS**

**Program Objective:** To solve an equation of the form \( f(x) = 0 \) by accelerated fixed point iteration (Aitkens' / Steffensen algorithm)

**Program Description:** The equation \( f(x) = 0 \) is solved implicitly for \( x \) to yield \( x = g(x) \). The function \( g(x) \) is the basis for the iteration:

\[
\begin{align*}
x_{i+1} &= x_i - \frac{(x_i - x_i^0)^2}{x_i - 2x_i + x_i^0} \\
x_i^0 &= g(x_i) \quad \text{and} \quad x_0^0 = g(x_0)
\end{align*}
\]

The resulting sequence of iterates converges quadratically to a unique fixed point on an interval \([a, b]\) if:

- \( g: [a, b] \rightarrow [a, b] \) and
- \( |g'(x)| < 1 \quad \forall x \in [a, b] \)

**User Instructions:**

1. Store the function \( g(x) \) in \( y_1 \) using \[ \text{\( y = \)} \]

2. Select the program STEFFENS for EXECution using \[ \text{PRGM} \]

3. Upon prompting, user supplies:
- \( X: \) the initial guess \( x_0 \),
- \( T \) for tolerance \( 10^{-1} \) and
- \( N \) for the maximum number of iterations

**Platform:** Texas Instruments Graphing Calculator TI83/84 (plus)

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**Program Listing**

```pascal
:Prompt X,T,N
:ClrHome
:For(I,1,N,1)
:y_1(X) - U
:y_1(U) - V
:X -(U-X)^2/(V-2U+X) - W
:If(abs(W-X) < 10^-T)
:Then
:Output(5,1,"X=")
:Output(5,3,Round(W,T))
:Output(6,1,"ITNS=")
:Output(6,6,I)
:Stop
:Else
:W - X
:End
:End
:Output(5,1,"TOL NOT MET IN")
:Output(6,6,N)
:Output(7,1,"ITNS=")
:Stop
```

**Command Location, remarks**

- PRGM I/O
- PRGM I/O
- PRGM Ctl
- VARS YVARS FUNCTION 1 , STO
- PRGM Ctl 2nd TEST
- PRGM Ctl
- PRGM I/O
- PRGM I/O , Math NUM
- PRGM I/O
- PRGM I/O
- PRGM I/O
- PRGM I/O
- PRGM Ctl (to end If)
- PRGM Ctl (to end For)
- PRGM Ctl
- PRGM Ctl