**PROGRAM**: NEWTON  
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**PURPOSE**: To compute the value of a zero (root) of a function $f(x)$.  
**PLATFORM**: Texas Instruments TI 82/83 graphing calculator

**DEFINITION**: $x = r$ is a root or zero of a function $f(x)$ means that $f(r) = 0$. The program implements Newton's method to approximate the root of a function $f(x)$. The derivative is approximated by a centered difference (CDQ):

$$x_{\text{new}} = x_{\text{old}} - \frac{f(x_{\text{old}})}{f'(x_{\text{old}})} \approx x_{\text{old}} - \frac{f(x_{\text{old}} + h) - f(x_{\text{old}} - h)}{2h}$$

which is applicable to any continuous, differentiable function. On the TI calculator, the CDQ is available as the nDeriv command.

**Input**: The function $f(x)$ is stored in $y_1$ prior to running the program.

**Output**: the approximate value of the root, stored in $A$

**Note**: For the program to successfully determine the value of a root of $f(x)$ make sure that:  
- the function actually has a root by graphing $f(x)$  
- and that both the function and its derivative exist on the interval of interest. Particularly, vertical slopes can be a problem.

**PROGRAM NEWTON**

```
:Prompt X
:X-y_1(X)/nDeriv(y_1(X),X,X)→A
:While Abs(X-A)>1EE-8
:A→X
:X-y_1(X)/nDeriv(y_1(X),X,X)→A
:Disp A
:End
:Stop
```

**Command Locations**

- Prompt: prgm I/O  
- Sto: Sto  
- While: Prgm Ctl  
- nDeriv: Math 8  
- Disp: Prgm Ctl  
- End: Prgm Ctl  
- Stop: Prgm Ctl

**EXAMPLE**: To solve the equation $3^x = 5x$ for $x$, we need to determine the root of the function $f(x) = 3^x - 5x$. (Answers: start with $x=0$: 0.26866911 and start with $x=2$: 2.17027659. The equation has two solutions.

**EXAMPLE**: To solve the equation $x^4 - 4x^2 = x - 2$ for $x$, we need to determine the root of the function $f(x) = x^4 - 4x^2 - x + 2$, (check graphically that there are four distinct real roots!).  
start with $x=-2$: -1.65121076  
start with $x=-1$: -1 (bingo!)  
start with $x=0$: 2  
start with $x=2$: 0.61803399. The equation has four solutions.