Introduction to the MAPLE Computer Algebra System.
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Part 5: Differentiation and Integration.

The Instantaneous Rate of Change (IROC) of a function at a point \( x = a \):

\[
IROC(a) = \lim_{{h \to 0}} \frac{{f(a + h) - f(a)}}{h}
\]

Taken at any point \( x \), the IROC(\( x \)) is often referred to as the derivative (function) of \( f \), written as \( f'(x) \). \( f'(x) = \lim_{{h \to 0}} \frac{{f(x + h) - f(x)}}{h} \).

Differentiation is the process of obtaining the derivative of a function.

Constructing a function's derivative from its definition.

The instantaneous rate of change (or derivative) of a function \( f \) at a point \( x = a \) is straightforward to compute from the above definition. For example, the derivative of \( f(x) = x^2 - 3x + 2 \) at \( x = 2 \) would be:

\[
\begin{align*}
> f &:= x \mapsto x^2 - 3x + 2; \\
> \text{limit}((-f(2+h)-f(2))/h, h=0);
\end{align*}
\]

The derivative (function) of \( f \), written as \( f'(x) \) can be obtained, too:

\[
\begin{align*}
> f' &:= \text{limit}((-f(x+h)-f(x))/h, h=0); \\
\end{align*}
\]

Since taking the derivative is a frequent task, the above command has been pre-programmed as **diff**.

The syntax is **diff(expression, variable name)**, the output is again an **expression**: \( f'(x) = \text{diff}(f(x), x) \);

Higher derivatives are available using the $ sign after the variable name:

\[
\begin{align*}
> f'' &:= \text{diff}(f(x), x$2$); \\
\end{align*}
\]

We can graph the function along with its derivatives:

\[
\begin{align*}
> \text{plot}([f(x), f', f''], x=-1..4, \text{thickness}=2); \\
\end{align*}
\]

In addition to the diff command, Maple has the D operator which produces a function as its output:

\[
\begin{align*}
> f' &:= \text{D}(f); \\
\end{align*}
\]

This allows for derivative values to be computed by function evaluation

\[
\begin{align*}
> f'(2); \\
\end{align*}
\]

The composition operator is used for higher derivatives:

\[
\begin{align*}
> f &:= x \mapsto \sin(2x); \\
> f'' &:= \text{D}(\text{D}@@3)(f); \\
\end{align*}
\]

To create a sequence (list) of successive derivatives:

\[
\begin{align*}
> L1 &:= [\text{seq}((\text{D}@n)(f)(x), n=0..5)]; \\
\end{align*}
\]

which can be plotted as:

\[
\begin{align*}
> \text{plot}(L1, x=0..2*\text{Pi}); \\
\end{align*}
\]