A rational function $f$ is defined as a quotient of polynomials $P$ and $Q$:

$$f(x) = \frac{P(x)}{Q(x)}$$

**Examples:**

The following are rational functions:  
$$f(x) = \frac{1}{x}, \frac{x}{x^2 - 1}, \frac{x^2}{x^2 - 1}, \frac{x^3}{x^2 - 1}$$

where these are not:

$$f(x) = x^2 - 1, \quad \frac{1}{\sqrt{x}}, \quad \frac{x}{x^2 - 1}, \quad \frac{|x|}{x^2 - 1}$$

**Sketching the Graph of a Rational Function.**

To sketch the graph of a rational function, we investigate the following characteristics:

i. **x-intercept:** set $y = 0$, then solve for $x$

$$y = 0 \implies f(x) = 0 \implies \frac{P(x)}{Q(x)} = 0, \text{ so } P(x) = 0 \text{ and } Q(x) \neq 0$$

ii. **y-intercept:** set $x = 0$ and substitute into $f$

$$x = 0 \implies y = f(0)$$

iii. **Vertical Asymptote (VA):** Set $Q(x) = 0$ and $P(x) \neq 0$ and solve for $x$

solve for $x$ \hspace{1cm} $Q(x) = 0$ \text{ and } $P(x) \neq 0$

iv. **Horizontal Asymptote (HA):** Consider the degree (highest power) of $P$ and $Q$

a. $\text{deg}(P) < \text{deg}(Q)$: $\text{HA}: y = 0$ (the x-axis)

b. $\text{deg}(P) = \text{deg}(Q)$: $\text{HA}: y = a/b$ (the quotient of the leading coefficients)

c. $\text{deg}(P) > \text{deg}(Q)$: no HA

v. **Calculate some additional function values:**

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
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<tbody>
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</table>

Type $y=$ to store the function in $y_1$ and quit

Then type $\text{vars}$, select $Y-VARS$, FUNCTION and 1

vi. **Confirm the graph features using the graph facility on your calculator**
1. Consider the function \( f(x) = \frac{2x - 1}{x^2 - 2} \). Compute the following characteristics of \( f \) (be sure to state the equation of the intercepts and asymptotes e.g. \( y = 3 \) - not just \( 3 \)). Then sketch the graph of \( f \).

a. Compute the \( x \)-intercept(s) of \( f \).

\( \quad \)  

b. Compute the \( y \)-intercept(s) of \( f \).

\( \quad \)  

c. Identify any vertical asymptotes.

\( V.A.: \quad \)  

d. Identify any horizontal asymptotes.

\( H.A.: \quad \)  

e. Sketch the graph of \( f \) in the figure:
2. Consider the function \( g(x) = \frac{x^2 + 1}{2x^2 - 1} \). Compute the following characteristics of \( g \) (be sure to state the equation of the intercepts and asymptotes). Then sketch the graph of \( g \).

a. Compute the \( x \)-intercept(s) of \( g \).

b. Compute the \( y \)-intercept(s) of \( g \).

c. Identify any vertical asymptotes.

V.A.: ______________

d. Identify any horizontal asymptotes.

H.A.: ______________

e. Sketch the graph of \( g \) in the figure.
3. Consider the function \[ h(x) = \frac{1 - 2x}{x^2 + 2}. \] Compute the following characteristics of \( h \) (be sure to state the equation of the intercepts and asymptotes). Then sketch the graph of \( h \).

a. Compute the \( x \)-intercept(s) of \( h \).

b. Compute the \( y \)-intercept(s) of \( h \).

c. Identify any vertical asymptotes.

V.A.: ______________

d. Identify any horizontal asymptotes.

H.A.: ______________

e. Sketch the graph of \( h \) in the adjacent figure.
4. Compare the functions \( f(x) = \frac{x - 2}{x^2 + 1} \) and \( g(x) = \frac{x + 2}{x^2 - 1} \) in terms of their intercepts and asymptotes:
(Be sure to state the equation of the intercepts and asymptotes)

<table>
<thead>
<tr>
<th></th>
<th>( f(x) )</th>
<th>( g(x) )</th>
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<tbody>
<tr>
<td>( x )-intercept(s)</td>
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<tr>
<td>( y )-intercept(s)</td>
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<tr>
<td>Vertical Asymptote(s)</td>
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<tr>
<td>Horizontal Asymptote(s)</td>
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</tbody>
</table>

5. Construct an expression for the function \( f(x) \) that has the following characteristics:
- the \( x \)-intercepts of \( f \) are \( x = -1 \) and \( x = 1 \).
- the \( y \)-intercept of \( f \) is \( y = 1 \).
- the vertical asymptotes of \( f \) are \( x = -2 \) and \( x = 2 \).
- the horizontal asymptote of \( f \) is \( y = 5/4 \).

\[ f(x) = \]