

# Bonds

## Definition

Long-term Debt: legally enforceable promise by borrower to pay specified amount(s) on specified date(s)

## Contractual Characteristics

Maturity (N)

Par/Face/Maturity Value (FV)

Coupon Rate, Coupon Payment (PMT)

$$\text{Coupon Payment} = \text{Coupon Rate} \times \text{Par Value}$$

Security and Seniority of claim

Call Provisions: Call Premium/Price, Date of First Call/Protection Period, Sinking Fund

Conversion Terms: Ratio, Exercise/Strike Price, Expiration

Indexing (e.g., TIPS)

## Risks

Default Risk

Interest Rate Risk (Price Risk, Risk to Wealth)

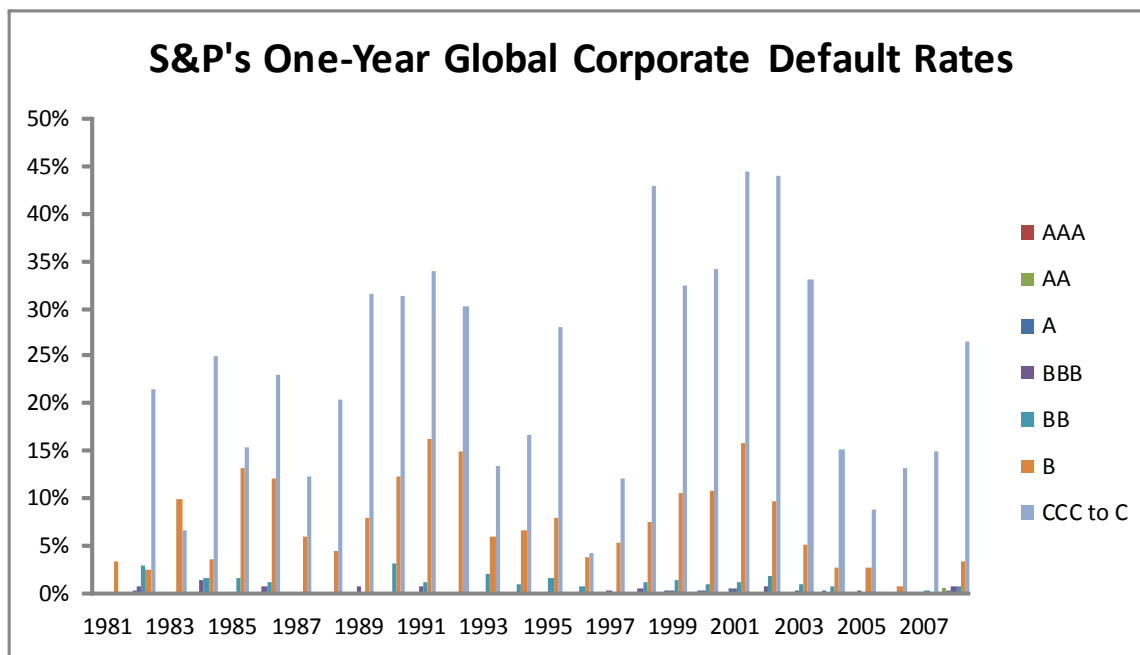
Reinvestment Rate Risk (Risk to Income)

# Default Risk and Bond Ratings

## Simplified Bond Ratings

|         | Investment Grade |    |   |     | Junk |   |     |   |
|---------|------------------|----|---|-----|------|---|-----|---|
| Moody's | Aaa              | Aa | A | Baa | Ba   | B | Caa | C |
| S&P     | AAA              | AA | A | BBB | BB   | B | CCC | D |

## Actual Default Rates



# Digression: Liquidation (“Chapter 7”)

## Priority of Payment

**Debt** owned by secured creditors (from sales of secured assets)

Trustee's costs

Wages (subject to limits)

Taxes

Unfunded pension liabilities

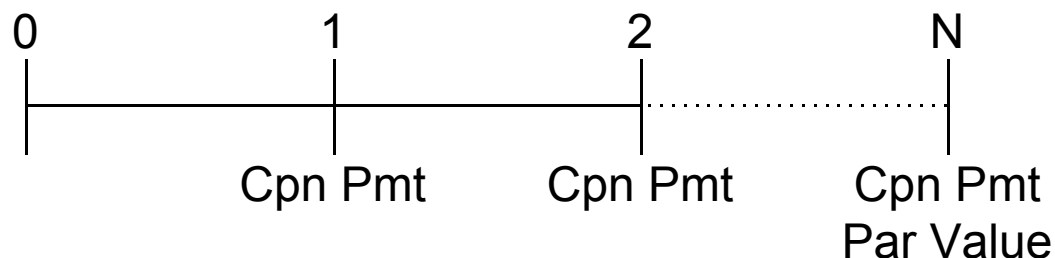
**Debt** owned by unsecured creditors (senior first, then junior)

Preferred stock

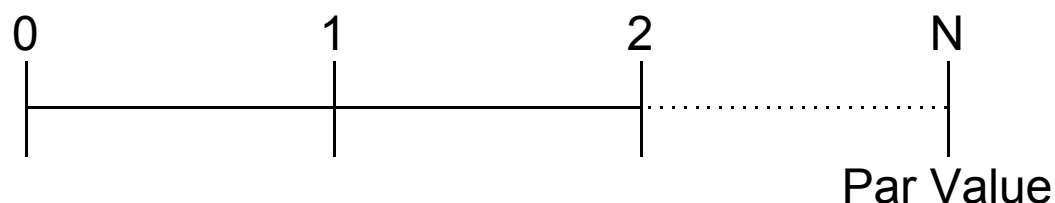
Common stock

# Bonds' Cash Flow Patterns

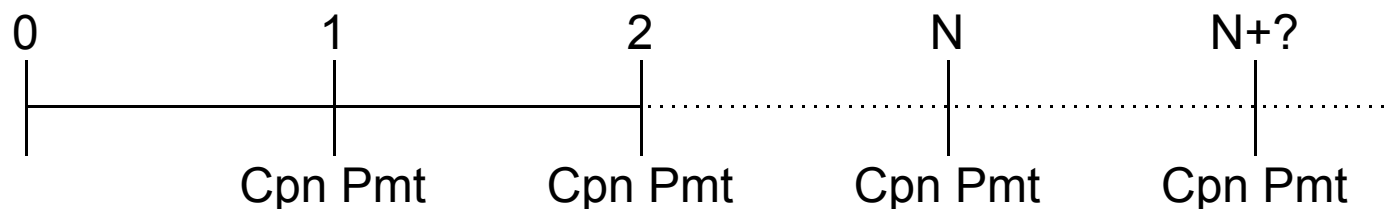
## Coupon Bond



## Zero-Coupon Bond, including U.S. Treasury STRIPS

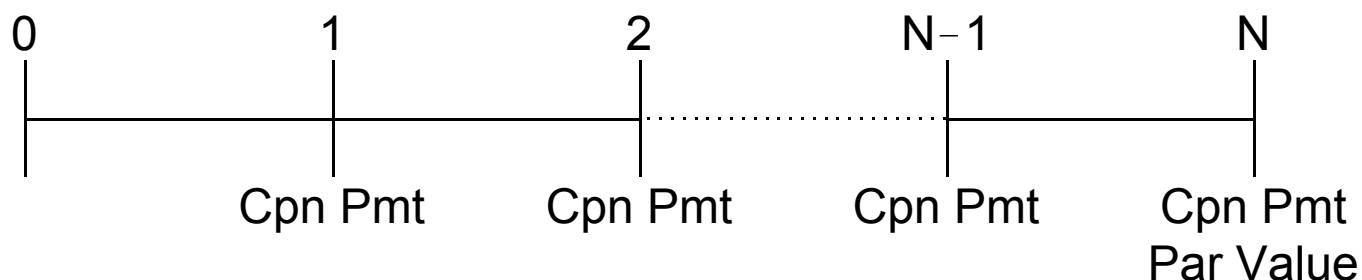


## Perpetual Bond, e.g., "Consol"



# Treasury STRIPS

**STRIPS originate as ordinary coupon bonds**



## Stripping

DIY zeros: If the parts are worth more than the whole, buy bond, detach coupons and corpus, sell separately. Each separate piece becomes a new zero-coupon bond (an FV with its own maturity).

## Reconstitution

If the whole is worth more than the parts, buy up parts, rebuild bond, sell.

## Valuation

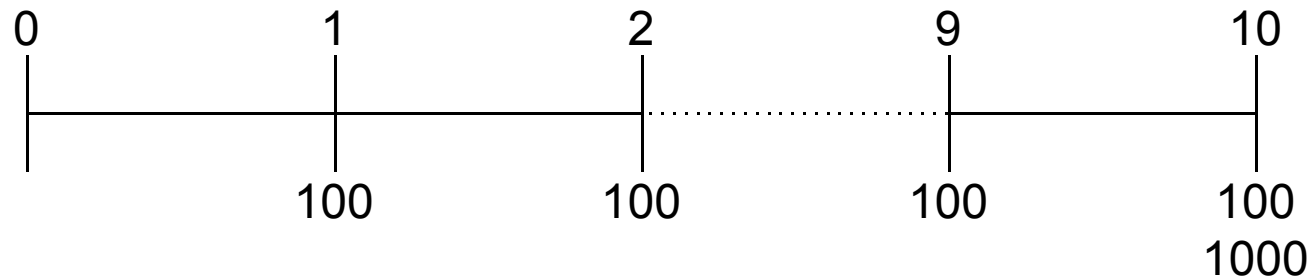
Because most other bonds make semiannual coupon payments (and even STRIPS start that way), value STRIPS as if they were semiannual, too.

# Treasury Inflation Protected Securities (TIPS)

## Purpose

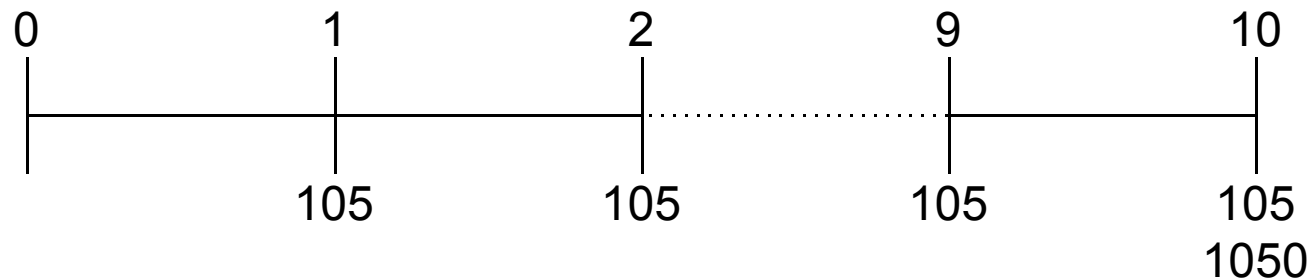
Discover inflation rate that *investors* expect

## Cash Flows (initial for TIPS, always for ordinary Treasuries)



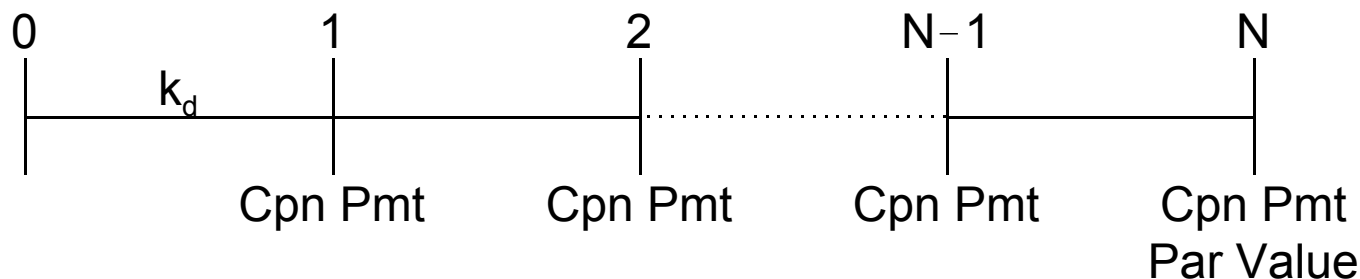
## Cash Flows (Adjusted)

If the price level rises by 5 percent, par value increases by same amount:



# Bond Valuation: Coupon Bond

## Cash Flows



## Value

$$\hat{P}_0 = \sum_{t=1}^N \frac{Cpn\ Pmt}{(1+k_d)^t} + \frac{Par\ Value}{(1+k_d)^N}$$

where

$$Cpn\ Pmt = Coupon\ Rate \times Par\ Value$$

$$k_{di} = k_{RF} + DRP_i + LP_i + MRP_i$$

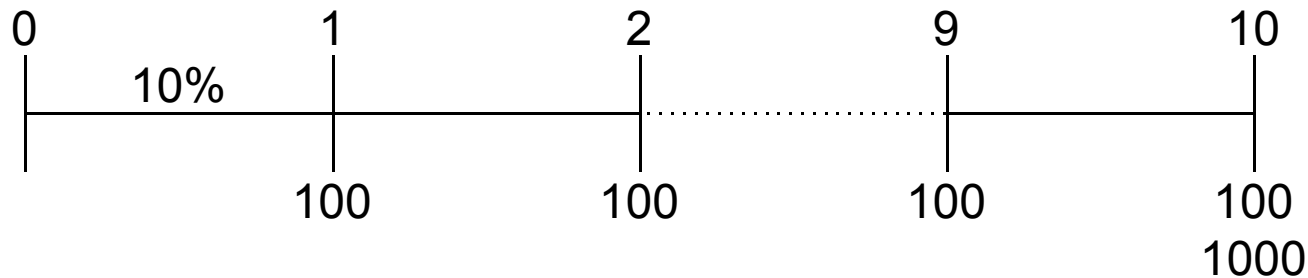
|         |     |   |              |
|---------|-----|---|--------------|
| Inputs: | N   | = | N            |
|         | I   | = | $k_d$        |
|         | PMT | = | Cpn Pmt      |
|         | FV  | = | Par Value    |
| Output: | PV  | = | $-\hat{P}_0$ |

# Coupon Bond Valuation: Practice 1

## Example

Consider a 10-year bond with a \$1,000 par value and a 10% annual coupon. If the interest rate is 10%, what's it worth?

## Cash Flows



## Value

$$\hat{P}_0 = \sum_{t=1}^{10} \frac{100}{(1+0.10)^t} + \frac{1000}{(1+0.10)^{10}}$$

## Calculator Solution

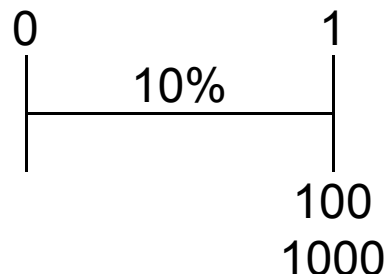
|         |     |   |      |
|---------|-----|---|------|
| Inputs: | N   | = | 10   |
|         | I   | = | 10   |
|         | PMT | = | 100  |
|         | FV  | = | 1000 |
| Output: | PV  | = |      |

# Coupon Bond Valuation: Practice 2

## Example

Consider a 1-year bond with a \$1,000 par and a 10% annual coupon. If the interest rate is 10%, what's it worth?

## Cash Flows



## Value

$$\hat{P}_0 = \sum_{t=1}^1 \frac{100}{(1+0.10)^t} + \frac{1000}{(1+0.10)^1}$$

## Calculator Solution

|         |     |   |      |
|---------|-----|---|------|
| Inputs: | N   | = | 1    |
|         | I   | = | 10   |
|         | PMT | = | 100  |
|         | FV  | = | 1000 |
| Output: | PV  | = |      |

# Interest Rates, Maturity and Bond Values: Practice

**Suppose the Interest Rate Increases to 13%**

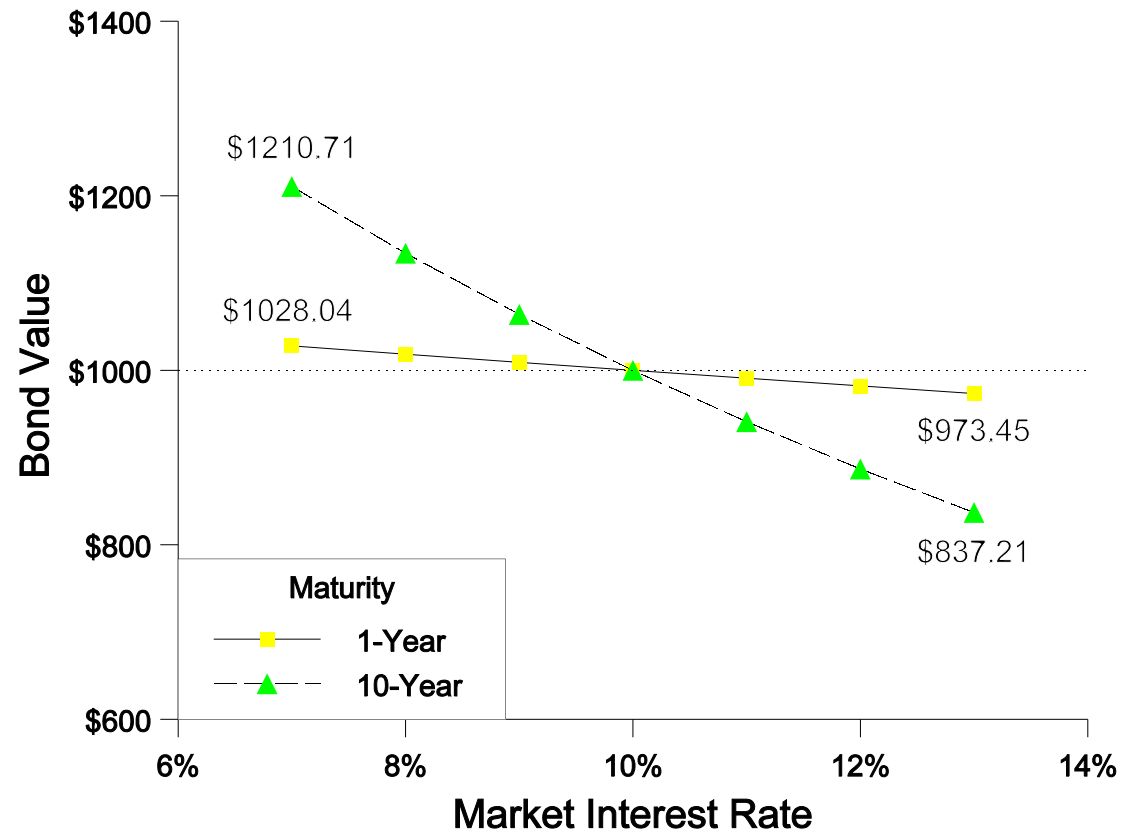
| Bond    |     |   | Long | Short |
|---------|-----|---|------|-------|
| Inputs: | N   | = | 10   | 1     |
|         | I   | = | 13   | 13    |
|         | PMT | = | 100  | 100   |
|         | FV  | = | 1000 | 1000  |
| Output: | PV  | = |      |       |

**Suppose the Interest Rate Decreases to 7%**

| Bond    |     |   | Long | Short |
|---------|-----|---|------|-------|
| Inputs: | N   | = | 10   | 1     |
|         | I   | = | 7    | 7     |
|         | PMT | = | 100  | 100   |
|         | FV  | = | 1000 | 1000  |
| Output: | PV  | = |      |       |

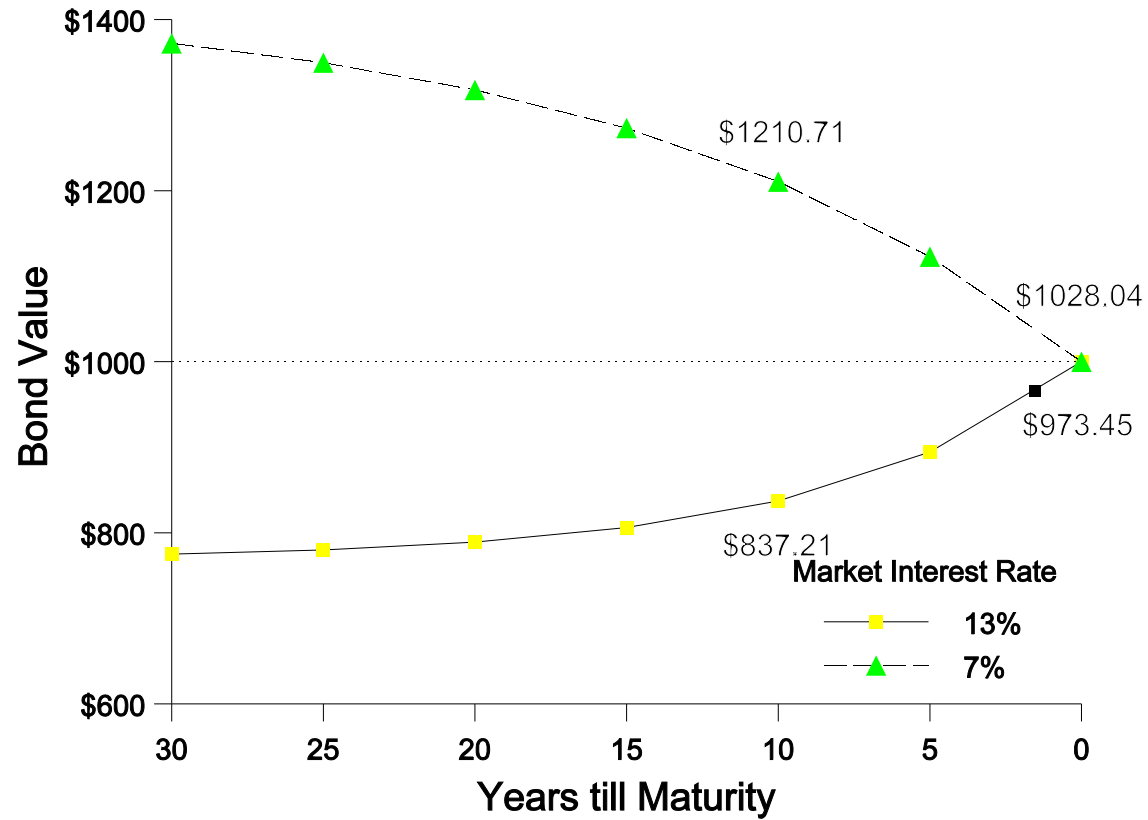
# Interest Rates and Bond Value

## Given Maturity



# Maturity and Bond Value

Given the Interest Rate



# Important Price-Yield Relationships

1. Bond values and interest rates are inversely related.

$$2. \quad \text{Market Rate} \begin{cases} < \\ = \\ > \end{cases} \text{Coupon Rate} \Leftrightarrow \text{Value} \begin{cases} > \\ = \\ < \end{cases} \text{Par}$$

When issuing new bonds, firms customarily set their coupon rate to equal the prevailing market interest rate, so that the new bonds will sell at par (also customarily set to 1000).

3. Interest rate risk is greater  
the longer the maturity,  
the lower the coupon rate, and  
the lower the initial yield.

Because of interest rate risk, long-term bonds usually carry higher rates than otherwise-identical short-term bonds.

Duration indicates the impact of all three elements simultaneously.

4. As a bond approaches maturity, its value approaches par.

# Interest Rate and Reinvestment Rate Risk: Comparison

## Direction of Effect

When market interest rates change:

- wealth changes in the opposite direction
- reinvested income changes in the same direction
- net effect depends on relative sizes of the two effects, which depends on maturity

## Exposure Depends on Bond's Characteristics

| Characteristic    | Risk to Wealth | Risk to Income |
|-------------------|----------------|----------------|
| Maturity: Long    | High           | Low            |
| Maturity: Short   | Low            | High           |
| Coupon Rate: Low  | High           | Low            |
| Coupon Rate: High | Low            | High           |

## Immunization

Construct portfolios of bonds with maturities and coupon rates, so that their gain/loss due to changes in reinvestment rates offset loss/gain due to price changes. (Use bonds with duration equal to investment horizon, rebalanced periodically.)

# Yield to Maturity (YTM)

## Definition

That discount rate which makes the PV of the promised CFs equal to the bond's price, i.e.,

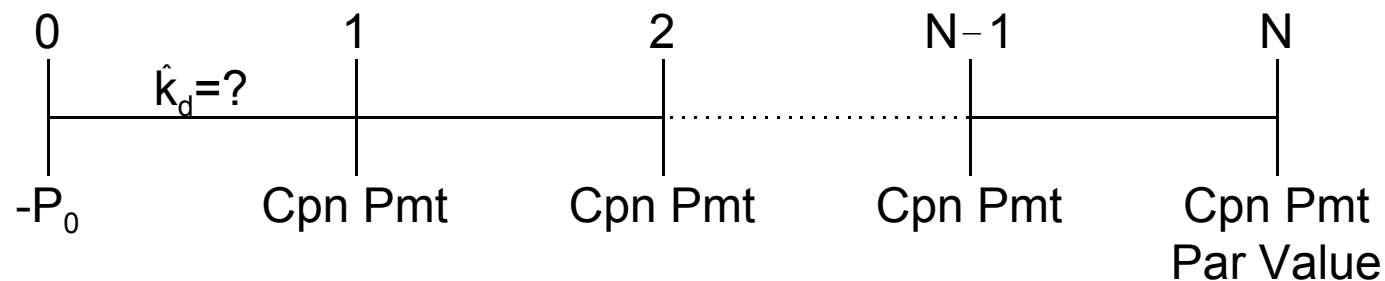
$$P_0 = \sum_{t=1}^N \frac{Cpn\ Pmt}{(1+\hat{k}_d)^t} + \frac{Par\ Value}{(1+\hat{k}_d)^N}$$

## Interpretation

Promised return and, in equilibrium, *required* return or market rate; a total return

## Illustration

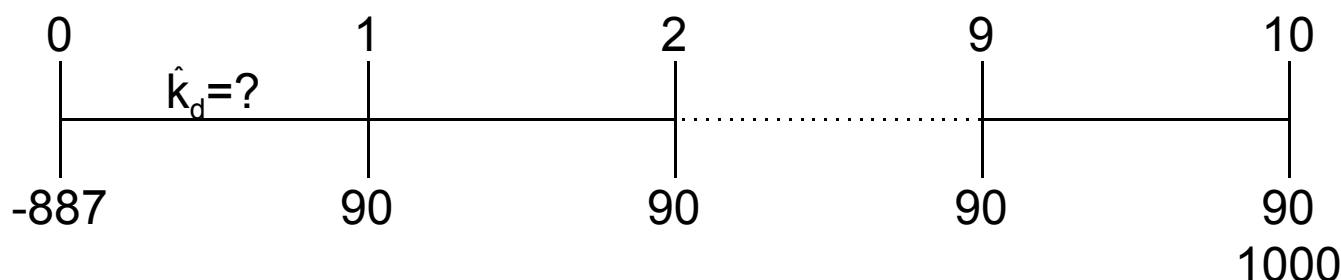
If a bank offered the same cash flows (as the bond) for the same price (deposit), what rate of return is implied?



# YTM: Practice

## Discount Bond

A 9% coupon, 10-year, \$1,000 par bond sells for \$887. Find its YTM. Is it consistent with price-yield relationships?



## Premium Bond

The same bond sells for \$1,134.20. Find its YTM. Is it consistent with price-yield relationships?

## Calculator Solution

|         |     |   |      |         |
|---------|-----|---|------|---------|
| Inputs: | N   | = | 10   | 10      |
|         | PV  | = | -887 | -1134.2 |
|         | PMT | = | 90   | 90      |
|         | FV  | = | 1000 | 1000    |
| Output: | I   | = |      |         |

# Current, Capital Gains, and Total Yields

## Current Yield (CY)

Annual coupon payment, expressed as a percentage of bond's current price:

$$CY_1 = \frac{Cpn\ Pmt}{P_0} \times 100$$

## Capital Gains Yield (CGY)

Annual change in bond's price (at a given interest rate), expressed as a percentage of its current price:

$$CGY_1 = \frac{\hat{P}_1 - P_0}{P_0} \times 100$$

## Total Yield (TY)

Coupon payments plus capital gain, expressed as a percentage of bond's current price. In equilibrium, equal to YTM.

$$TY_1 = \frac{Cpn\ Pmt + (\hat{P}_1 - P_0)}{P_0} \times 100 = CY_1 + CGY_1$$

# Current, Capital Gains, and Total Yields: Practice

## Discount Bond

A 9% coupon, 10-year, \$1,000 par bond sells for \$887. Find its Current Yield, Capital Gains Yield and Total Yield.

## Premium Bond

The same bond sells for \$1,134.20. Find its Current Yield, Capital Gains Yield and Total Yield.

## Calculator Solution

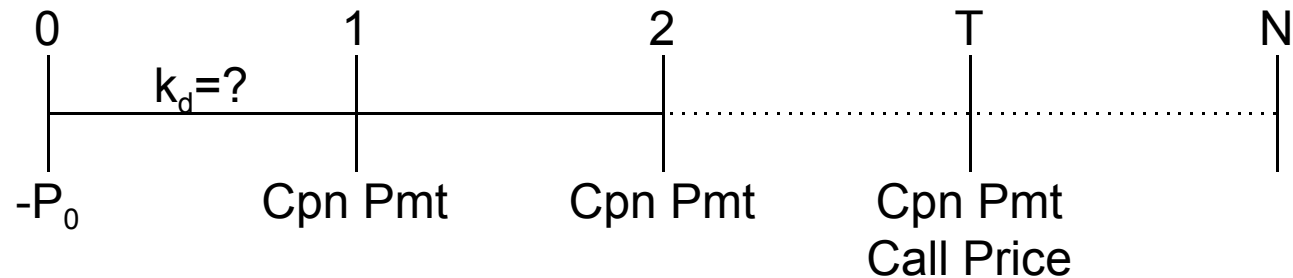
|             |   |     |        |
|-------------|---|-----|--------|
| Cpn Pmt     | = | 90  | 90     |
| $P_0$       | = | 887 | 1134.2 |
| $CY_1$      | = |     |        |
| $\hat{P}_1$ | = |     |        |
| $CGY_1$     | = |     |        |
| $TY_1$      | = |     |        |

# Yield to First Call (YTC)

## Concept

Since a bond may be called before its maturity, a bondholder can't count on earning the YTM. How much can he earn, if the bond is called as soon as it becomes eligible for call?

## Cash Flows, if Called



## Calculation

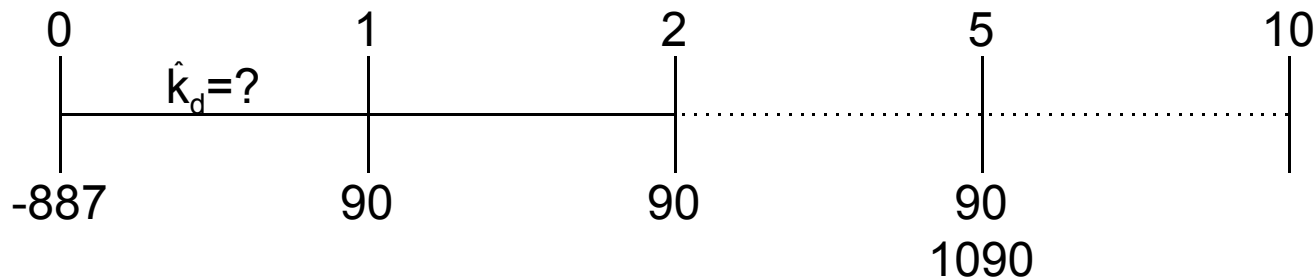
Like calculating YTM, after substituting date of first call (T) for maturity (N) and Call Price for Par Value

$$P_0 = \sum_{t=1}^T \frac{Cpn\ Pmt}{(1+\hat{k}_d)^t} + \frac{Call\ Price}{(1+\hat{k}_d)^T}$$

# YTC: Practice

## Discount Bond

A 9% coupon, 10-year, \$1,000 par bond selling for \$887, is callable in 5 years at a call price of \$1,090. Find its YTC.



## Premium Bond

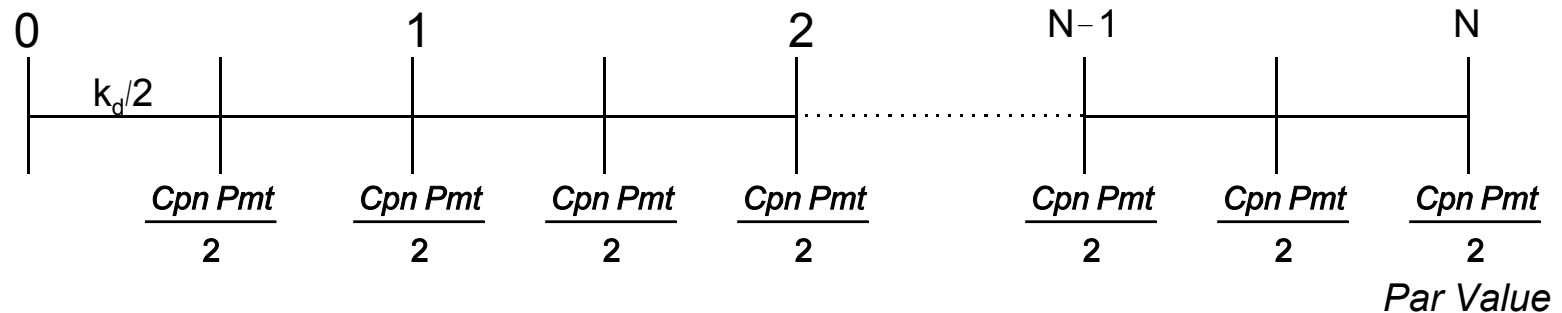
The same bond sells for \$1,134.20. Find its YTC.

## Calculator Solution

|         |     |   |      |         |
|---------|-----|---|------|---------|
| Inputs: | N   | = | 5    | 5       |
|         | PV  | = | -887 | -1134.2 |
|         | PMT | = | 90   | 90      |
|         | FV  | = | 1090 | 1090    |
| Output: | I   | = |      |         |

# Semiannual Coupons

## Cash Flows



## Value

$$P_0 = \sum_{t=1}^{2N} \frac{\frac{Cpn\ Pmt}{2}}{\left(1 + \frac{k_d}{2}\right)^t} + \frac{Par\ Value}{\left(1 + \frac{k_d}{2}\right)^{2N}}$$

## Calculator Solution

|         |     |   |                      |
|---------|-----|---|----------------------|
| Inputs: | N   | = | 2N                   |
|         | I   | = | $k_d/2$              |
|         | PMT | = | $\frac{Cpn\ Pmt}{2}$ |
| Output: | FV  | = | Par Value            |
|         | PV  | = | $-\hat{P}_0$         |

# Semiannual Coupons: Value (Practice)

**Interest Rate = 10%**

Bond

Inputs:

N =  
I =  
PMT =  
FV =

Long

|      |
|------|
| 20   |
| 5    |
| 50   |
| 1000 |
|      |

Short

|      |
|------|
| 2    |
| 5    |
| 50   |
| 1000 |
|      |

Output:

PV =

**Interest Rate = 13%**

Bond

Inputs:

N =  
I =  
PMT =  
FV =

Long

|      |
|------|
| 20   |
| 6.5  |
| 50   |
| 1000 |
|      |

Short

|      |
|------|
| 2    |
| 6.5  |
| 50   |
| 1000 |
|      |

Output:

PV =

**Interest Rate = 7%**

Bond

Inputs:

N =  
I =  
PMT =  
FV =

Long

|      |
|------|
| 20   |
| 3.5  |
| 50   |
| 1000 |
|      |

Short

|      |
|------|
| 2    |
| 3.5  |
| 50   |
| 1000 |
|      |

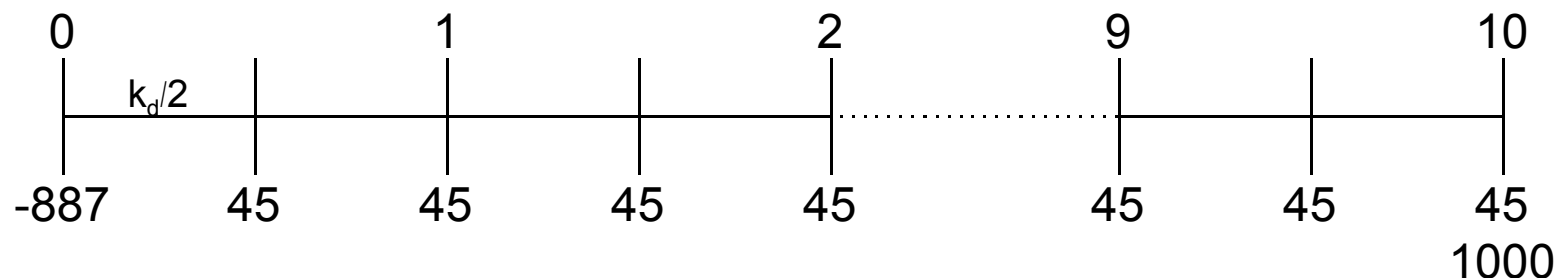
Output:

PV =

# Semiannual Coupons: YTM (Practice)

## Discount Bond

A 9% semiannual-coupon, 10-year, \$1,000 par bond sells for \$887. Find its YTM. Is it consistent with price-yield relationships?



## Premium Bond

The same bond sells for \$1,134.20. Find its YTM. Is it consistent with price-yield relationships?

## Calculator Solution

|         |     |   |      |         |
|---------|-----|---|------|---------|
| Inputs: | N   | = | 20   | 20      |
|         | PV  | = | -887 | -1134.2 |
|         | PMT | = | 45   | 45      |
|         | FV  | = | 1000 | 1000    |
| Output: | I   | = |      |         |

Similarly for YTC.