

## Bonds

### Definition

Long-term Debt: legally enforceable promise by borrower to pay specified amount(s) on specified date(s)

### Contractual Characteristics

Maturity (N)  
Par/Face/Maturity Value (FV)  
Coupon Rate, Coupon Payment (PMT)

$$\text{Coupon Payment} = \text{Coupon Rate} \times \text{Par Value}$$

Security and Seniority of claim  
Call Provisions: Call Premium/Price, Date of First Call/Protection Period, Sinking Fund  
Conversion Terms: Ratio, Exercise/Strike Price, Expiration  
Indexing (e.g., TIPS)

### Risks

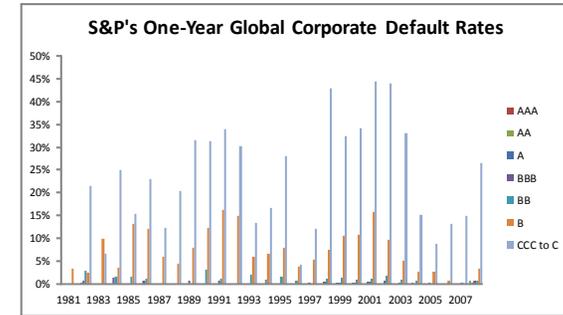
Default Risk  
Interest Rate Risk (Price Risk, Risk to Wealth)  
Reinvestment Rate Risk (Risk to Income)

## Default Risk and Bond Ratings

### Simplified Bond Ratings

	Investment Grade				Junk			
Moody's	Aaa	Aa	A	Baa	Ba	B	Caa	C
S&P	AAA	AA	A	BBB	BB	B	CCC	D

### Actual Default Rates



## Digression: Liquidation ("Chapter 7")

### Priority of Payment

**Debt** owned by secured creditors (from sales of secured assets)

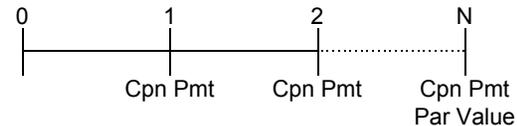
Trustee's costs  
Wages (subject to limits)  
Taxes  
Unfunded pension liabilities

**Debt** owned by unsecured creditors (senior first, then junior)

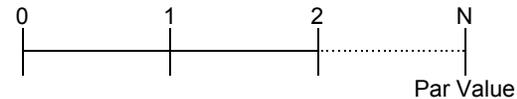
Preferred stock  
Common stock

## Bonds' Cash Flow Patterns

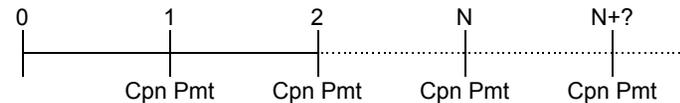
### Coupon Bond



### Zero-Coupon Bond, including U.S. Treasury STRIPS

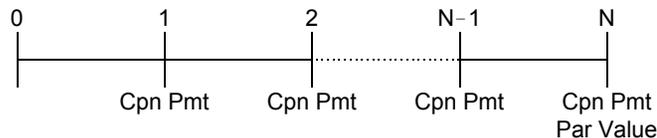


### Perpetual Bond, e.g., "Consol"



## Treasury STRIPS

STRIPS originate as ordinary coupon bonds



### Stripping

DIY zeros: If the parts are worth more than the whole, buy bond, detach coupons and corpus, sell separately. Each separate piece becomes a new zero-coupon bond (an FV with its own maturity).

### Reconstitution

If the whole is worth more than the parts, buy up parts, rebuild bond, sell.

### Valuation

Because most other bonds make semiannual coupon payments (and even STRIPS start that way), value STRIPS as if they were semiannual, too.

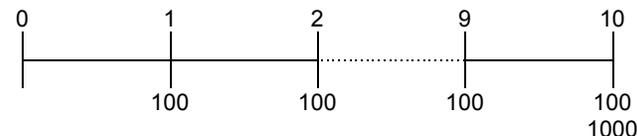
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## Treasury Inflation Protected Securities (TIPS)

### Purpose

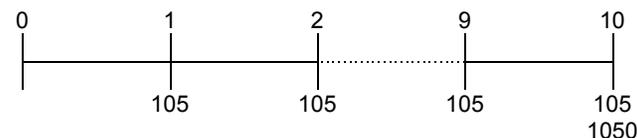
Discover inflation rate that *investors* expect

Cash Flows (initial for TIPS, always for ordinary Treasuries)



### Cash Flows (Adjusted)

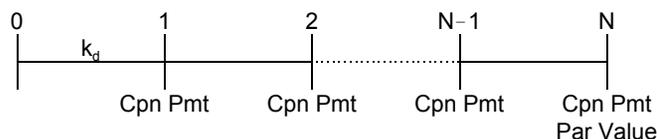
If the price level rises by 5 percent, par value increases by same amount:



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## Bond Valuation: Coupon Bond

### Cash Flows



### Value

$$\hat{P}_0 = \sum_{t=1}^N \frac{Cpn\ Pmt}{(1+k_d)^t} + \frac{Par\ Value}{(1+k_d)^N}$$

Inputs:	N =	N
	I =	$k_d$
	PMT =	Cpn Pmt
	FV =	Par Value
Output:	PV =	$-\hat{P}_0$

where

$$Cpn\ Pmt = Coupon\ Rate \times Par\ Value$$

$$k_{di} = k_{RF} + DRP_i + LP_i + MRP_i$$

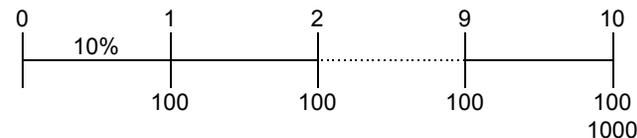
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## Coupon Bond Valuation: Practice 1

### Example

Consider a 10-year bond with a \$1,000 par value and a 10% annual coupon. If the interest rate is 10%, what's it worth?

### Cash Flows



### Value

$$\hat{P}_0 = \sum_{t=1}^{10} \frac{100}{(1+0.10)^t} + \frac{1000}{(1+0.10)^{10}}$$

### Calculator Solution

Inputs:	N =	10
	I =	10
	PMT =	100
	FV =	1000
Output:	PV =	

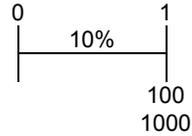
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## Coupon Bond Valuation: Practice 2

### Example

Consider a 1-year bond with a \$1,000 par and a 10% annual coupon. If the interest rate is 10%, what's it worth?

### Cash Flows



### Value

$$\hat{P}_0 = \sum_{t=1}^1 \frac{100}{(1+0.10)^t} + \frac{1000}{(1+0.10)^1}$$

### Calculator Solution

Inputs:	N =	1
	I =	10
	PMT =	100
	FV =	1000
Output:	PV =	

## Interest Rates, Maturity and Bond Values: Practice

### Suppose the Interest Rate Increases to 13%

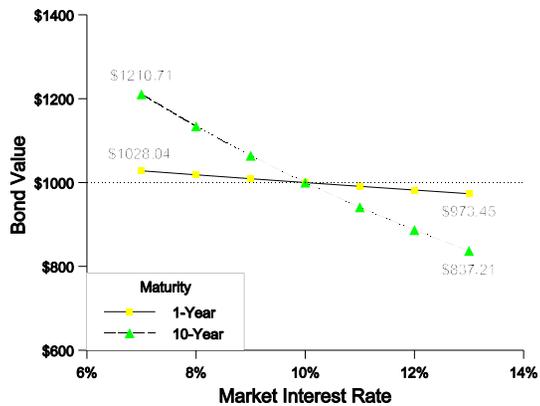
Bond		Long	Short
Inputs:	N =	10	1
	I =	13	13
	PMT =	100	100
	FV =	1000	1000
Output:	PV =		

### Suppose the Interest Rate Decreases to 7%

Bond		Long	Short
Inputs:	N =	10	1
	I =	7	7
	PMT =	100	100
	FV =	1000	1000
Output:	PV =		

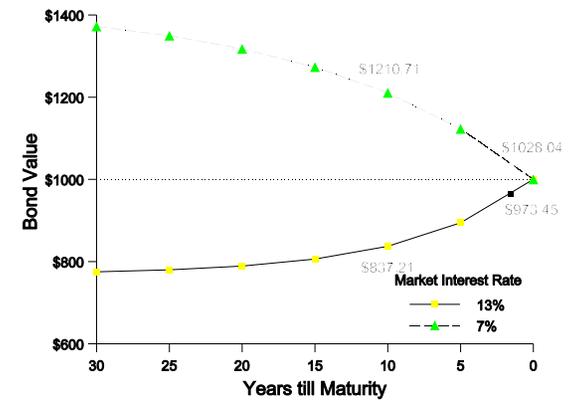
## Interest Rates and Bond Value

### Given Maturity



## Maturity and Bond Value

### Given the Interest Rate



## Important Price-Yield Relationships

1. Bond values and interest rates are inversely related.

$$2. \text{Market Rate} \begin{cases} < \\ = \\ > \end{cases} \text{Coupon Rate} \leftrightarrow \text{Value} \begin{cases} > \\ = \\ < \end{cases} \text{Par}$$

When issuing new bonds, firms customarily set their coupon rate to equal the prevailing market interest rate, so that the new bonds will sell at par (also customarily set to 1000).

3. Interest rate risk is greater the longer the maturity, the lower the coupon rate, and the lower the initial yield.

Because of interest rate risk, long-term bonds usually carry higher rates than otherwise-identical short-term bonds.

Duration indicates the impact of all three elements simultaneously.

4. As a bond approaches maturity, its value approaches par.

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## Interest Rate and Reinvestment Rate Risk: Comparison

### Direction of Effect

When market interest rates change:

- wealth changes in the opposite direction
- reinvested income changes in the same direction
- net effect depends on relative sizes of the two effects, which depends on maturity

### Exposure Depends on Bond's Characteristics

Characteristic	Risk to Wealth	Risk to Income
Maturity: Long	High	Low
Maturity: Short	Low	High
Coupon Rate: Low	High	Low
Coupon Rate: High	Low	High

### Immunization

Construct portfolios of bonds with maturities and coupon rates, so that their gain/loss due to changes in reinvestment rates offset loss/gain due to price changes. (Use bonds with duration equal to investment horizon, rebalanced periodically.)

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## Yield to Maturity (YTM)

### Definition

That discount rate which makes the PV of the promised CFs equal to the bond's price, i.e.,

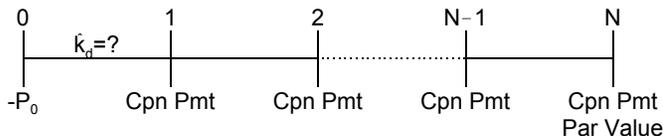
$$P_0 = \sum_{t=1}^N \frac{\text{Cpn Pmt}}{(1+\hat{k}_d)^t} + \frac{\text{Par Value}}{(1+\hat{k}_d)^N}$$

### Interpretation

Promised return and, in equilibrium, *required* return or market rate; a total return

### Illustration

If a bank offered the same cash flows (as the bond) for the same price (deposit), what rate of return is implied?

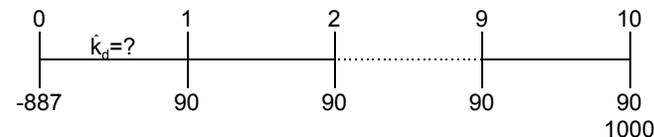


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## YTM: Practice

### Discount Bond

A 9% coupon, 10-year, \$1,000 par bond sells for \$887. Find its YTM. Is it consistent with price-yield relationships?



### Premium Bond

The same bond sells for \$1,134.20. Find its YTM. Is it consistent with price-yield relationships?

### Calculator Solution

Inputs:	N =	10	10
	PV =	-887	-1134.2
	PMT =	90	90
	FV =	1000	1000
Output:	I =		

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## Current, Capital Gains, and Total Yields

### Current Yield (CY)

Annual coupon payment, expressed as a percentage of bond's current price:

$$CY_1 = \frac{Cpn\ Pmt}{P_0} \times 100$$

### Capital Gains Yield (CGY)

Annual change in bond's price (at a given interest rate), expressed as a percentage of its current price:

$$CGY_1 = \frac{\hat{P}_1 - P_0}{P_0} \times 100$$

### Total Yield (TY)

Coupon payments plus capital gain, expressed as a percentage of bond's current price. In equilibrium, equal to YTM.

$$TY_1 = \frac{Cpn\ Pmt + (\hat{P}_1 - P_0)}{P_0} \times 100 = CY_1 + CGY_1$$

## Current, Capital Gains, and Total Yields: Practice

### Discount Bond

A 9% coupon, 10-year, \$1,000 par bond sells for \$887. Find its Current Yield, Capital Gains Yield and Total Yield.

### Premium Bond

The same bond sells for \$1,134.20. Find its Current Yield, Capital Gains Yield and Total Yield.

### Calculator Solution

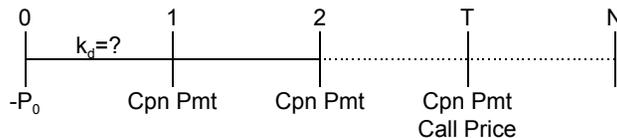
Cpn Pmt	=	90	90
$P_0$	=	887	1134.2
$CY_1$	=		
$\hat{P}_1$	=		
$CGY_1$	=		
$TY_1$	=		

## Yield to First Call (YTC)

### Concept

Since a bond may be called before its maturity, a bondholder can't count on earning the YTM. How much can he earn, if the bond is called as soon as it becomes eligible for call?

### Cash Flows, if Called



### Calculation

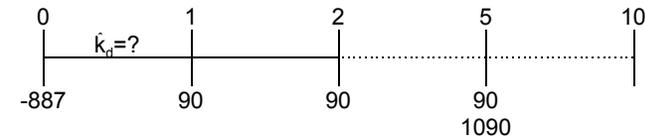
Like calculating YTM, after substituting date of first call (T) for maturity (N) and Call Price for Par Value

$$P_0 = \sum_{t=1}^T \frac{Cpn\ Pmt}{(1+\hat{k}_d)^t} + \frac{Call\ Price}{(1+\hat{k}_d)^T}$$

## YTC: Practice

### Discount Bond

A 9% coupon, 10-year, \$1,000 par bond selling for \$887, is callable in 5 years at a call price of \$1,090. Find its YTC.



### Premium Bond

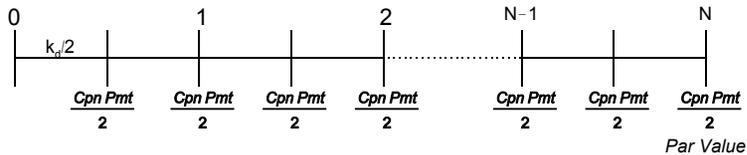
The same bond sells for \$1,134.20. Find its YTC.

### Calculator Solution

Inputs:	N	=	5	5
	PV	=	-887	-1134.2
	PMT	=	90	90
	FV	=	1090	1090
Output:	I	=		

## Semiannual Coupons

### Cash Flows



### Value

$$P_0 = \sum_{t=1}^{2N} \frac{\frac{Cpn\ Pmt}{2}}{\left(1 + \frac{k_d}{2}\right)^t} + \frac{Par\ Value}{\left(1 + \frac{k_d}{2}\right)^{2N}}$$

### Calculator Solution

Inputs:	N =	2N
	I =	$k_d/2$
	PMT =	Cpn Pmt/2
	FV =	Par Value
Output:	PV =	$-P_0$

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## Semiannual Coupons: Value (Practice)

Interest Rate = 10%	Bond Inputs:		Long	Short
		N =	20	2
		I =	5	5
		PMT =	50	50
		FV =	1000	1000
	Output:	PV =		

Interest Rate = 13%	Bond Inputs:		Long	Short
		N =	20	2
		I =	6.5	6.5
		PMT =	50	50
		FV =	1000	1000
	Output:	PV =		

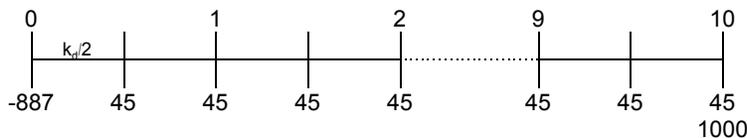
Interest Rate = 7%	Bond Inputs:		Long	Short
		N =	20	2
		I =	3.5	3.5
		PMT =	50	50
		FV =	1000	1000
	Output:	PV =		

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## Semiannual Coupons: YTM (Practice)

### Discount Bond

A 9% semiannual-coupon, 10-year, \$1,000 par bond sells for \$887. Find its YTM. Is it consistent with price-yield relationships?



### Premium Bond

The same bond sells for \$1,134.20. Find its YTM. Is it consistent with price-yield relationships?

### Calculator Solution

Inputs:	N =	20	20
	PV =	-887	-1134.2
	PMT =	45	45
	FV =	1000	1000
Output:	I =		

Similarly for YTC.

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