

Capital Budgeting

Definition

Planned Expenditure on Long-term Assets (including intangibles)

Issue

Which investment projects should be accepted?

Importance

Wealth Creation

Strategic—determines industry

Prolonged consequences

Survival and Growth

Large expenditures—outside financing

Purpose

Maximize shareholder wealth by investing in all projects which are *worth more than they cost*, i.e., for which

$$PV(\text{benefits}) > PV(\text{costs})$$

Project Analysis

Process

Parallel to security valuation:

1. Estimate project's cash flows
2. Estimate cash flows' risk and determine required return
3. Compare PVs of inflows and outflows
4. Accept project if $PV(\text{cash inflows}) > PV(\text{cash outflows})$

Note: These investment opportunities are often *created*

Other Uses for these Techniques

Mergers and Acquisitions

Spinoffs (Disinvestment)

Bond Refunding

Lease-Purchase Analysis

Investment Decision Rules

Requirements

To be worth using, a rule must:

- consider *all* of a project's cash flows and *only* its cash flows
- use appropriate discount rate (opportunity cost, from market)
- properly handle *mutually exclusive* projects

Mutual Exclusion

If two projects are independent, can take both

If two projects are mutually exclusive, must choose one (require ranking)

- May arise due to redundancy or capital rationing
- Very common

Selected Rules

Accounting (Book) Rate of Return

Payback Period

Discounted Payback Period

Net Present Value (NPV) ★

Profitability Index (PI)

Internal Rate of Return (IRR)

Modified Internal Rate of Return (MIRR)

Discounted-Cash-Flow (DCF) methods

Payback Period

$$0 = \sum_{t=0}^{PP} CF_t$$

Year	Expected Cash Flows		Cumulative Cash Flow	
	Project A	Project B	Project A	Project B
0	(10,000)	(10,000)	(10,000)	(10,000)
1	6,000	5,000	(4,000)	(5,000)
2	4,000	5,000	0	0
3	3,000		3,000	0
4	2,000		5,000	0
5	1,000		6,000	0

Discounted Payback Period

$$0 = \sum_{t=0}^{\text{DPP}} \frac{CF_t}{(1 + \text{MCC})^t}$$

MCC = 10%

Year	Expected Cash Flows		Cumulative PV	
	Project A	Project B	Project A	Project B
0	(10,000)	(10,000)	(10,000)	(10,000)
1	6,000	5,000	(4,545)	(5,455)
2	4,000	5,000	(1,240)	(1,322)
3	3,000		1,014	(1,322)
4	2,000		2,380	(1,322)
5	1,000		3,001	(1,322)

Net Present Value (NPV)

Definition

$$\mathbf{NPV} = PV(\text{cash inflows}) - PV(\text{cash outflows})$$

$$= \sum_{t=0}^n \frac{CF_t}{(1 + MCC)^t}$$

Decision Rule

Accept project if its $\mathbf{NPV} \geq 0$

Profitability Index (PI)

Definition

$$\begin{aligned} \mathbf{PI} &= \frac{\text{PV}(\text{cash inflows})}{\text{PV}(\text{cash outflows})} \\ &= \frac{\sum_{t=0}^n \frac{CIF_t}{(1+MCC)^t}}{\sum_{t=0}^n \frac{COF_t}{(1+MCC)^t}} \end{aligned}$$

Decision Rule

Accept project if its $PI \geq 1$

Internal Rate of Return (IRR)

Definition

IRR is that (hypothetical) discount rate which forces

$$PV(\text{cash outflows}) = PV(\text{cash inflows})$$

$$0 = NPV$$

$$0 = \sum_{t=0}^n \frac{CF_t}{(1 + \mathbf{IRR})^t}$$

Decision Rule

Accept project if its $IRR \geq MCC$

Note: Same as YTM

$$CF_0 = \sum_{t=1}^n \frac{CF_t}{(1 + \mathbf{IRR})^t}$$

Modified Internal Rate of Return (MIRR)

Definition

MIRR is that (hypothetical) discount rate which forces

PV (cash outflows) = PV (terminal value)

$$\sum_{t=0}^n \frac{COF_t}{(1 + \text{discount rate})^t} = \frac{\sum_{t=0}^n CIF_t (1 + \text{reinvestment rate})^{n-t}}{(1 + \mathbf{MIRR})^n}$$

Decision Rule

Accept project if its MIRR \geq MCC

Note: Can use different discount and reinvestment rates

Relationship between NPV, IRR and PI

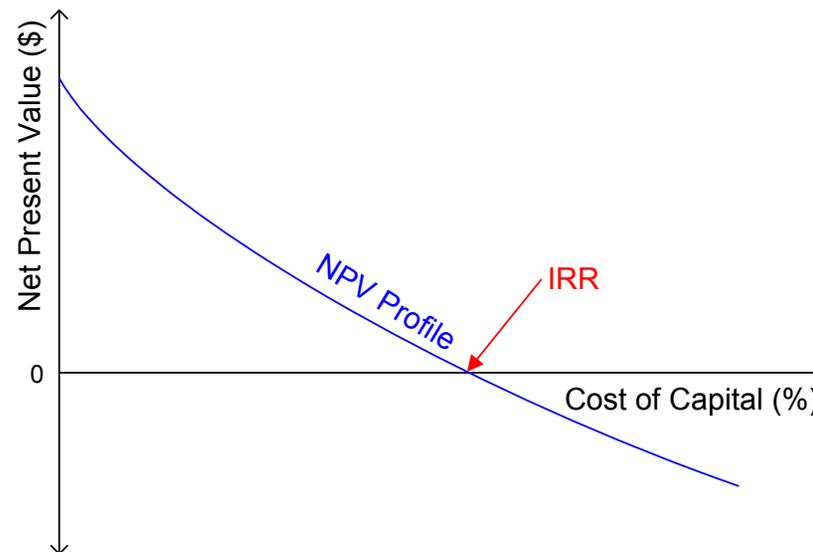
NPV and PI

If $NPV \geq 0$, then $PI \geq 1$

$$PI = \frac{NPV + PV(\text{cash outflows})}{PV(\text{cash outflows})}$$
$$= \frac{NPV}{PV(\text{cash outflows})} + 1$$

NPV and IRR

If $NPV \geq 0$, then $IRR \geq MCC$



Example Data

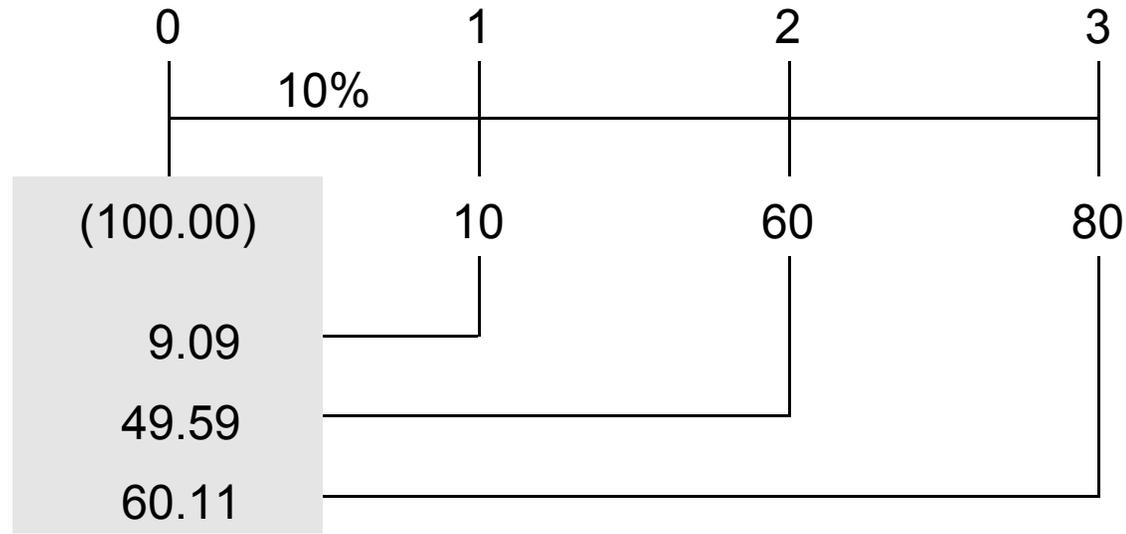
Marginal Cost of Capital (MCC) = 10%

	Expected Cash Flows	
Year	Project L	Project S
0	(100)	(100)
1	10	70
2	60	50
3	80	20

Fill in the blanks: evaluate the projects on this and the following pages.

Practice: NPV

Project L

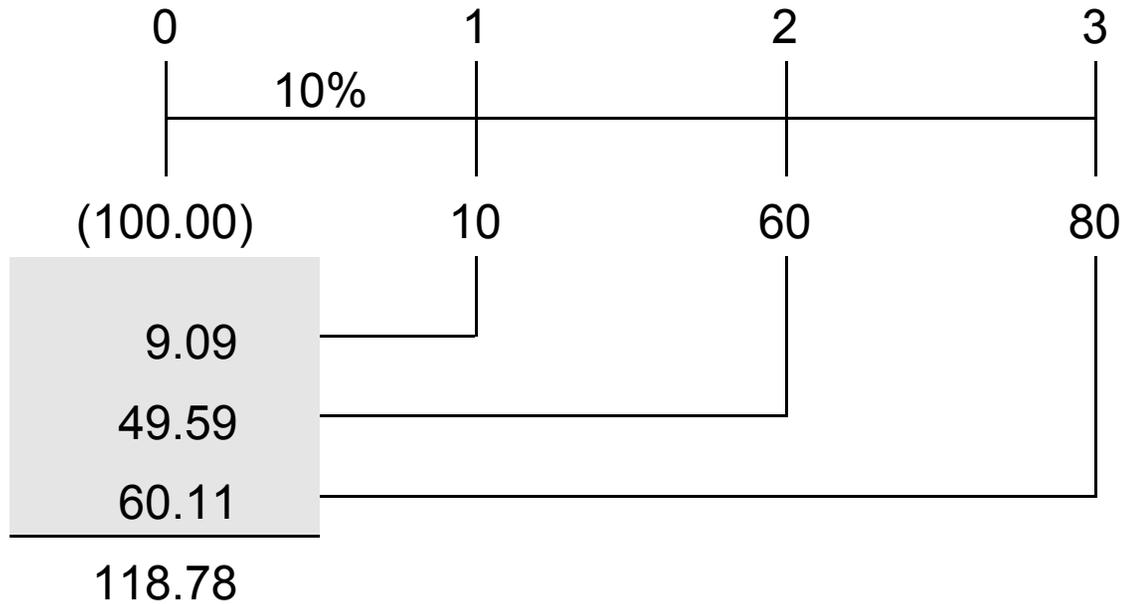


Decision

	Project L	Project S
NPV		
Accept?		

Practice: PI

Project L

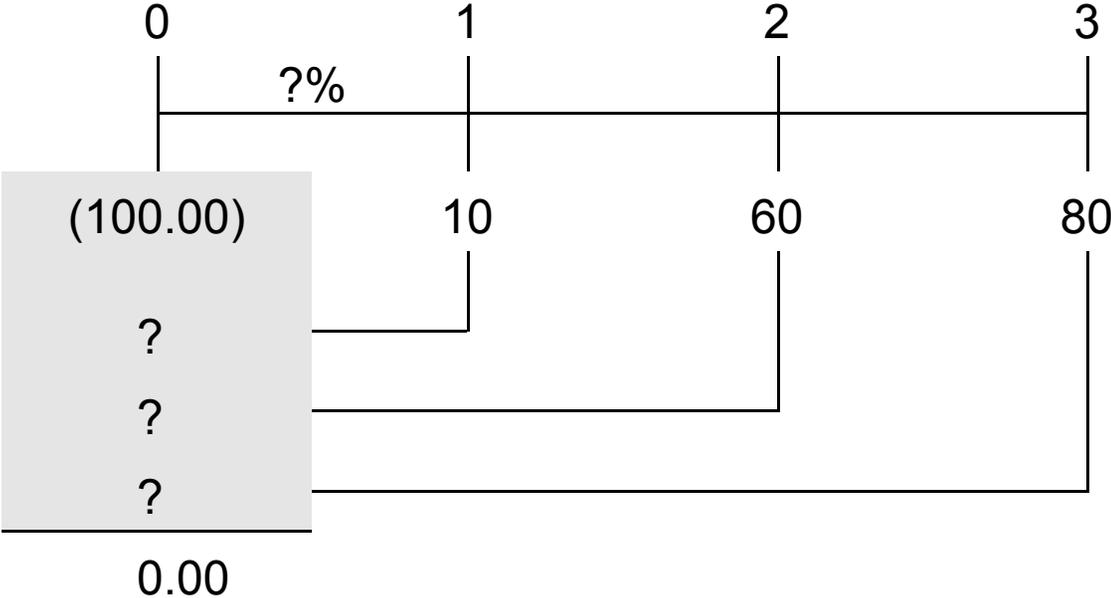


Decision

	Project L	Project S
PV (inflows)		
PV (outflows)		
PI		
Accept?		

Practice: IRR

Project L

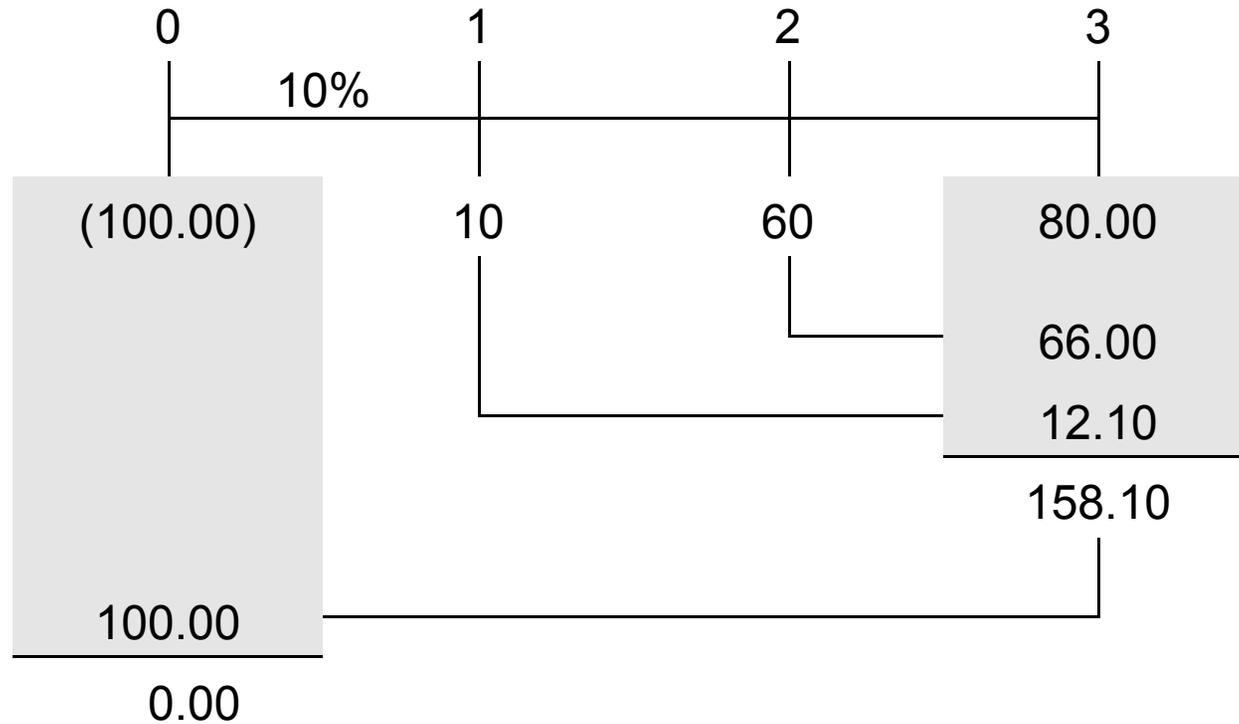


Decision

	Project L	Project S
IRR		
Accept?		

Practice: MIRR

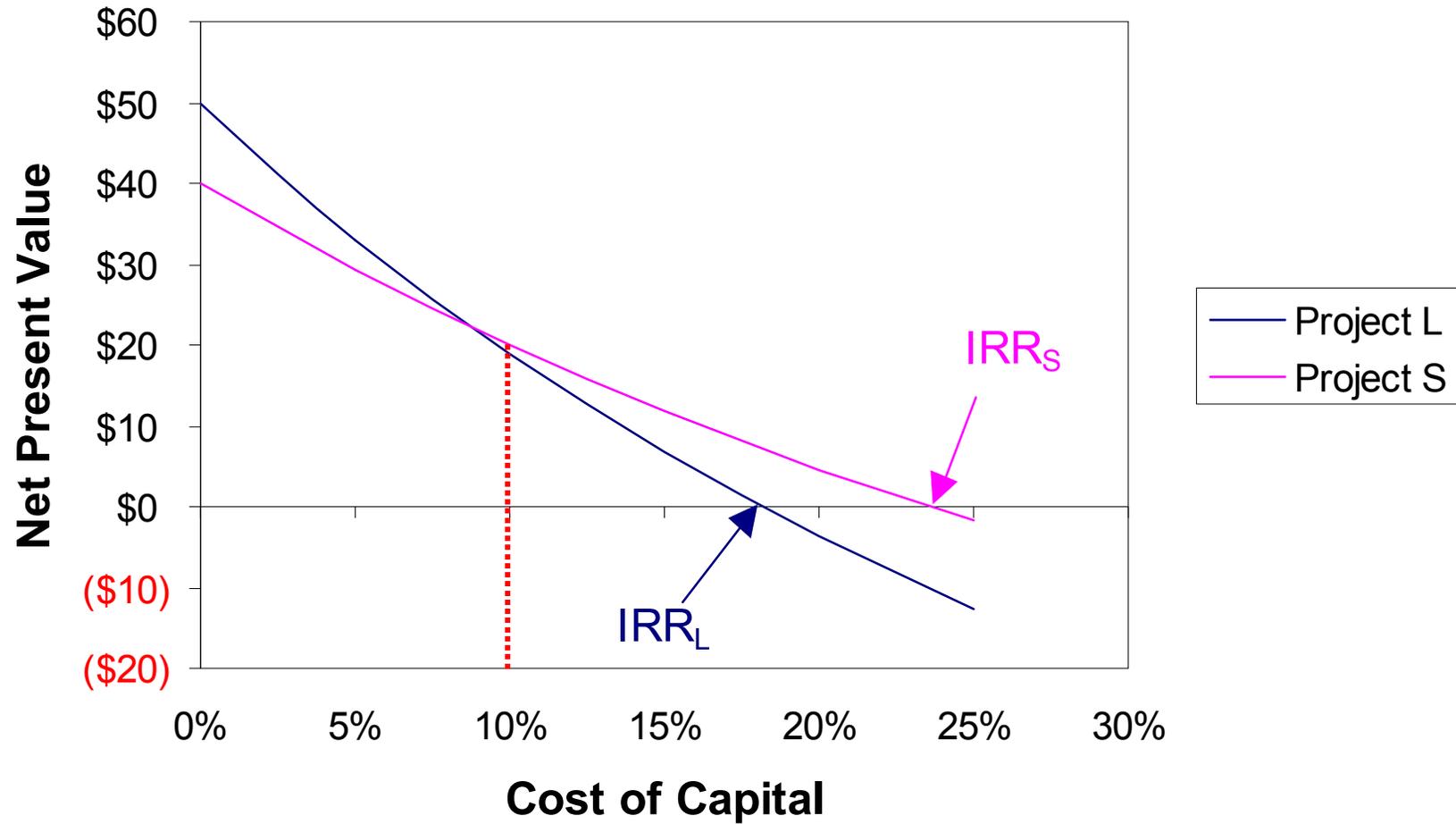
Project L



Decision

	Project L	Project S
PV (Costs)		
TV_3		
MIRR		
Accept?		

NPV Profiles



DCF Methods Are Not Independent

$$\text{NPV} \begin{cases} > \\ = \\ < \end{cases} 0 \Leftrightarrow \text{IRR} \begin{cases} > \\ = \\ < \end{cases} \text{MCC} \Leftrightarrow \text{MIRR} \begin{cases} > \\ = \\ < \end{cases} \text{MCC} \Leftrightarrow \text{PI} \begin{cases} > \\ = \\ < \end{cases} 1$$

Independent projects

- Good/Bad \Rightarrow Accept/Reject
- NPV, IRR, and PI give same decision

Mutually exclusive projects

- Good/Better \Rightarrow Rank
- NPV, IRR, and PI *may* give conflicting decisions, if projects differ in:

Scale	Size of investment
Timing	Early vs. late payoffs
Lives	Number of cash flows

Mutual Exclusion: Differences in Scale

MCC = 10%

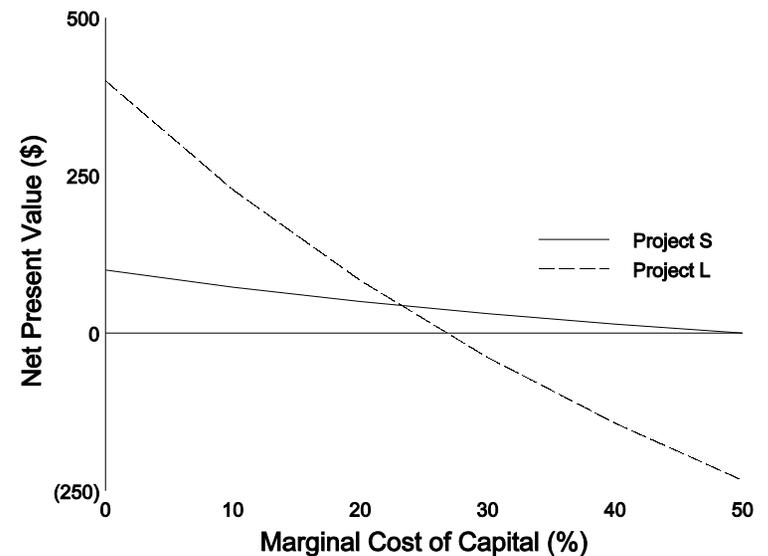
Year	Project S	Project L	Project Δ
0	(200)	(1,500)	(1,300)
1	300	1,900	1,600
NPV	\$72.73	\$227.27	\$154.55
PI	1.36	1.15	1.12
IRR	50.00%	26.67%	23.08%
MIRR	50.00%	26.67%	
Accept?			

Possible Conflict

Between: NPV v. others

Cause: absolute (\$) v. relative (%)
measures of benefit

Remedies: NPV, "Project Δ"



Mutual Exclusion: Differences in Timing

MCC = 10%

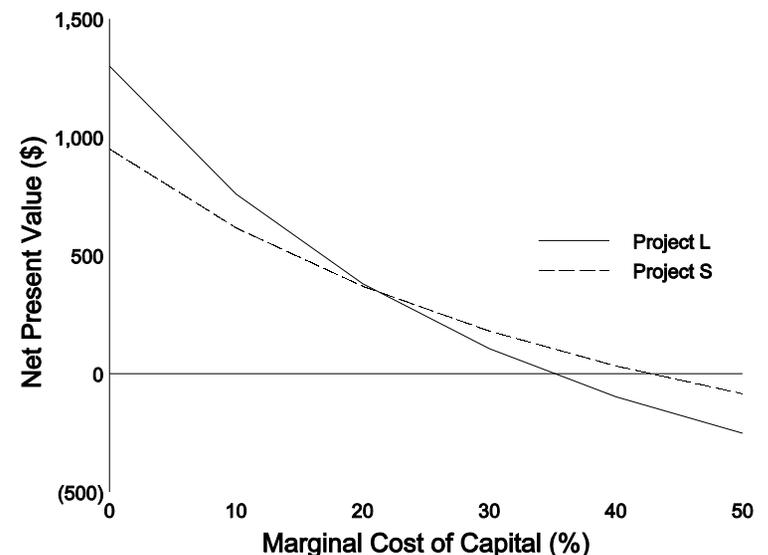
Year	Project L	Project S	Project Δ
0	(1,000)	(1,000)	0
1	100	650	(550)
2	200	650	(450)
3	2,000	650	1,350
NPV	\$758.83	\$616.45	\$142.37
PI	1.76	1.62	1.16
IRR	34.84%	42.57%	21.01%
MIRR	32.78%	29.10%	15.69%
Accept?			

Possible Conflict

Between: IRR v. others

Cause: different reinvestment rates
(MCC or IRR)

Remedies: NPV, "Project Δ," MIRR



Mutual Exclusion: Differences in Lives

MCC = 10%

Year	Project S	Project L
0	(1,000)	(1,000)
1	500	300
2	500	300
3	500	300
4		300
5		300
6		300
NPV	\$243.43	\$306.58
Accept?		

Possible Conflict

Between: NPV v. NPV

Cause: Different reinvestment opportunities

Remedies: Replacement Chain, Equivalent Annual Annuity

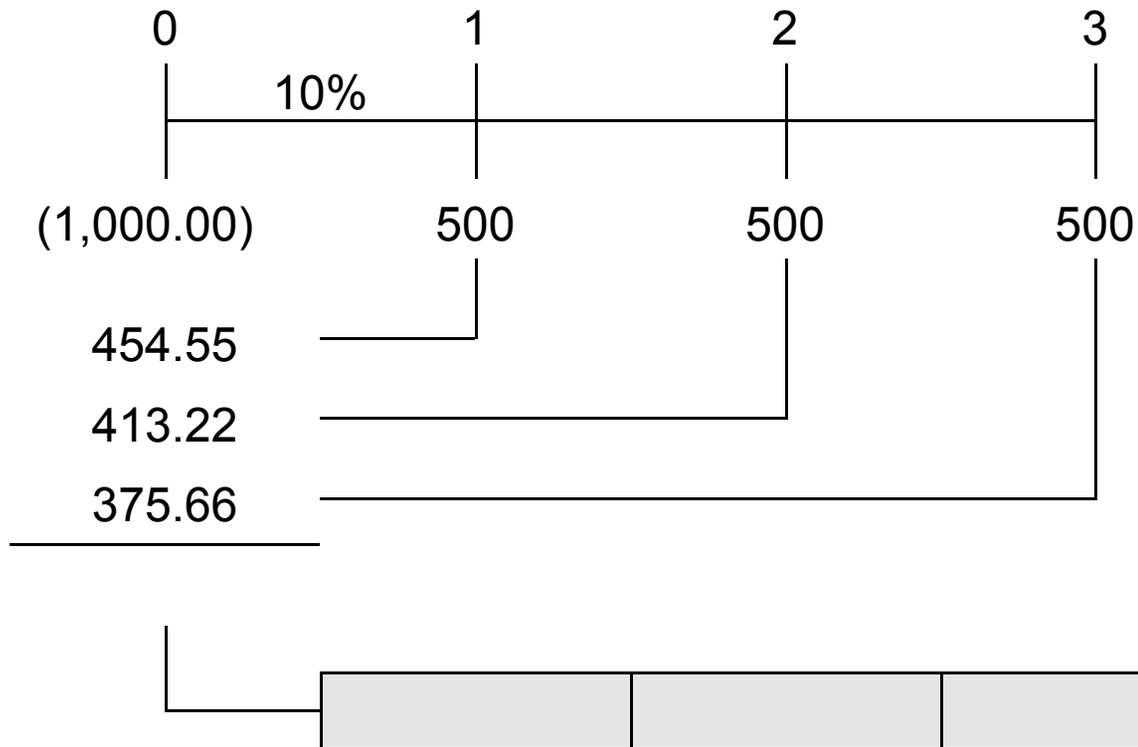
Different Lives: Replacement Chain

MCC = 10%

Year	Project S			Project L
	Original	Replacement	Combined	
0	(1,000)		(1,000)	(1,000)
1	500		500	300
2	500		500	300
3	500	(1,000)	(500)	300
4		500	500	300
5		500	500	300
6		500	500	300
NPV	\$243.43	\$182.89	\$426.32	\$306.58
Accept?				

Different Lives: Equivalent Annual Annuity (EAA)

Project S



Practice: EAA

MCC = 10%

Year	Project S	Project L
0	(1,000)	(1,000)
1	500	300
2	500	300
3	500	300
4		300
5		300
6		300
NPV	\$243.43	\$306.58
Life	3	6
EAA		
Accept?		

Summary: Ranking Mutually Exclusive Projects

Case	<i>Possible</i> Conflict	Cause	Remedies
Scale	NPV v. others	absolute v. relative measures	NPV Δ project
Timing	IRR v. others	different reinvestment rates	NPV Δ project MIRR
Lives	NPV v. NPV	different reinvestment opportunities	replacement chain EAA

Mutually Exclusive Projects: More Examples

[Click Here](#)

Other Problems with IRR: Non-normal Projects

Definitions

Normal: Cash outflows precede inflows (costs before benefits)

Non-normal: Any other cash flow pattern

Problem and Remedy

IRR misleading (or may not even exist)

Must *invert* IRR rule or use NPV

Example

MCC = 10%

Year	Investment	Financing	Investment	Financing
0	(100)	100	(130)	130
1	130	(130)	100	(100)
NPV	\$18.18	(\$18.18)	(\$39.09)	\$39.09
PI	1.18	0.85	0.70	1.43
IRR	30.00%	30.00%	(23.08%)	(23.08%)
MIRR	30.00%	(15.38%)	(23.08%)	43.00%

Other Problems with IRR: Multiple IRRs

Problem and Remedy

As many IRRs as sign changes in cash stream

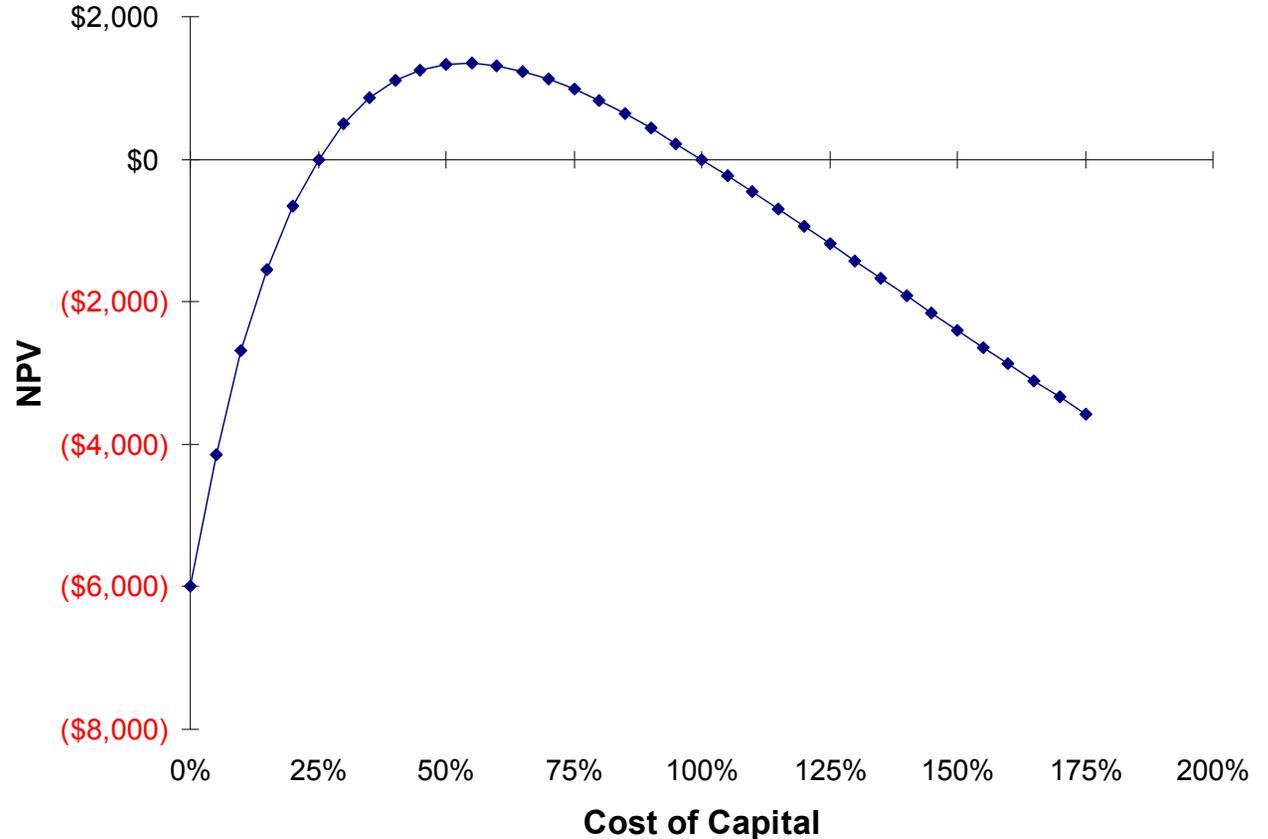
IRR irrelevant

Use NPV or MIRR

Example

MCC = 10%

Year	Cash Flow
0	(24,000)
1	78,000
2	(60,000)
NPV	(\$2,677.69)
PI	0.96
IRR	
MIRR	7.98%



Summary: IRR and Non-normal Cash Flows

Cash Flows	Cause	Remedy
inflows precede outflows	IRR a “cost”	invert IRR rule others
multiple sign changes	IRR a mathematical notion	others (IRR useless)

Capital Rationing

Definition

Dollar limit on capital budget

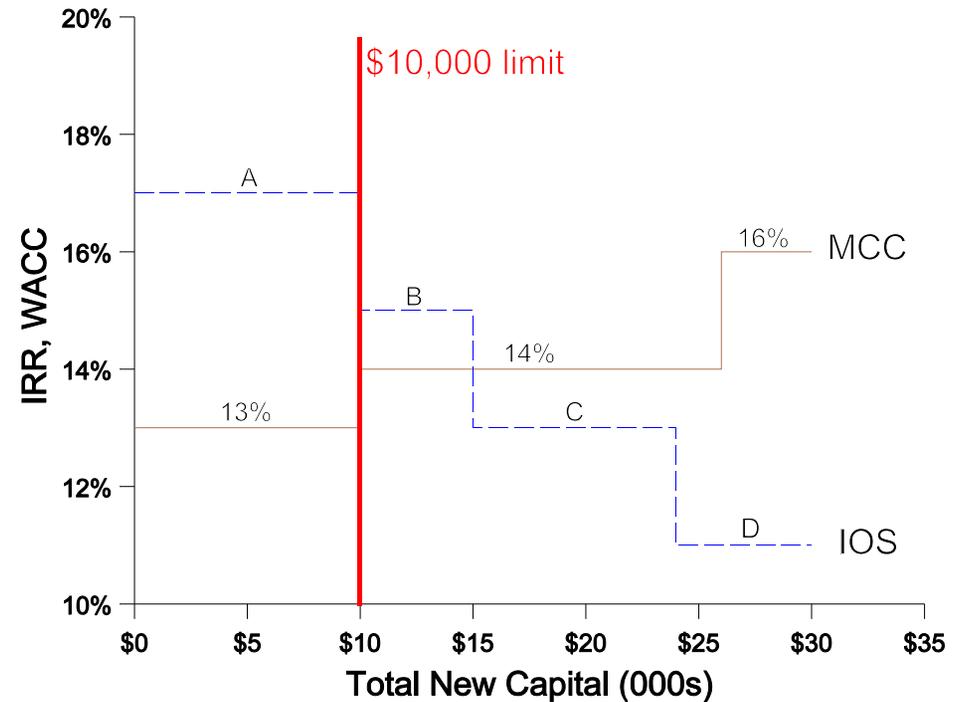
- Hard or Soft
- Binding or Non-binding

Problems

- If binding, violates NPV rule: can't max shareholder wealth because can't take all positive-NPV projects
- Creates mutual exclusion unnecessarily

Solution

1. Relax constraint
2. find group of projects with maximum *joint* NPV



Variable Cost of Capital

Issue

MCC expected to vary over future periods

Solution

Discount stepwise, one period at a time, starting with most distant cash flow

$$\text{NPV} = \sum_{t=0}^n \frac{CF_t}{\prod_{j=0}^t (1 + MCC_j)}$$

Mandated Investments

Scenario

Mutually exclusive projects, mandated by government
Pollution abatement

Problem

For mandated investments, NPVs may be negative

Solution

Take project with highest NPV (although negative)

Note: when projects' risks differ, riskier project evaluated using *lower* MCC

Present Value of Future Costs (PVC)

Scenario

Mutually exclusive projects, which differ only in costs (i.e., revenues unaffected by choice)
Common situation in replacement of equipment

Problem

If omit incremental revenues (which are unaffected), NPVs appear negative

Solution

Take project which has smallest cost, in present value terms

$$\mathbf{PVC} = \sum_{t=0}^n \frac{COF_t}{(1 + MCC)^t}$$

Note: when projects' risks differ, riskier project evaluated using *lower* MCC