Risk and Return

Issue

What is investor's *required rate of return*, i.e., the minimum rate for which he's willing to buy/hold asset?

Importance

Discount rate for valuation, investment decisions

Definitions

Return: annual percentage change in wealth, e.g., $\frac{D_1 + (P_1 - P_0)}{P_0}$ for common stock

Risk: variability of returns, deviation of actual outcome from expectation

Investors' Attitudes

"Rational" Investors Prefer

More Return (Greedy)

Less Risk (Risk-Averse)

- *will not bear more risk without earning more return*
- will pay (or accept less return) in order to avoid risk

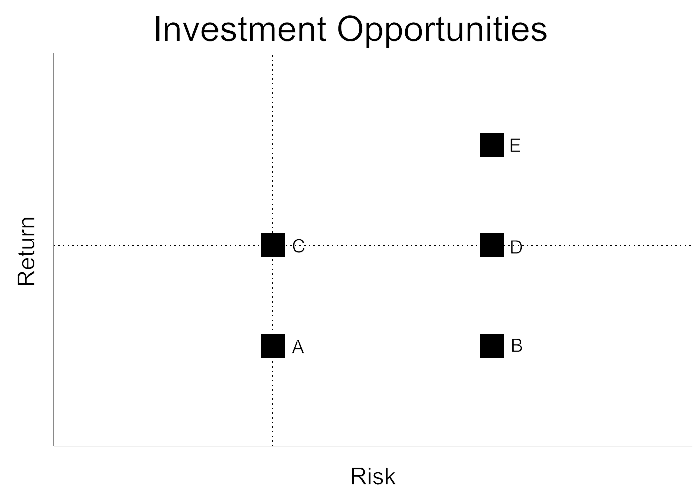
Choices

Rank potential investments by comparing

- expected returns (given risks)
- risks (given expected returns)

Set of non-dominated assets is called "efficient"

Rational investors are interested only in efficient assets



Risk-Return Tradeoff

Can't earn more return without bearing more risk

Required Rate of Return

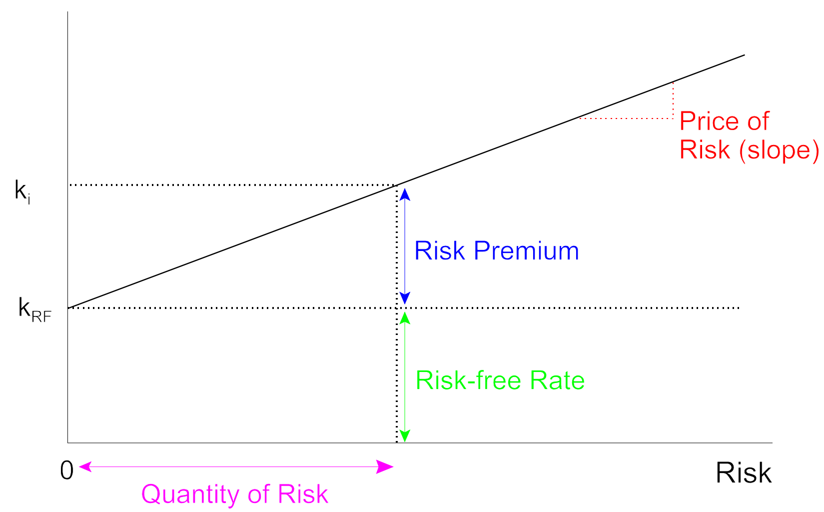
Required Rate of Return

$$k_i = \text{Risk-free Rate} + \text{Risk Premium for Asset } i$$
$$= \quad k_{RF} \quad + \quad RP_i$$

Risk Premium

Extra return required to bear a given amount of risk: $RP_i = \begin{pmatrix}\text{Quantity}\\ \text{of}\\ \text{Risk}\end{pmatrix}_i \times \begin{pmatrix}\text{Price}\\ \text{of}\\ \text{Risk}\end{pmatrix}$

All assets pay the risk-free rate.
Risky assets also pay a risk premium.
The riskier the asset, the larger the premium.

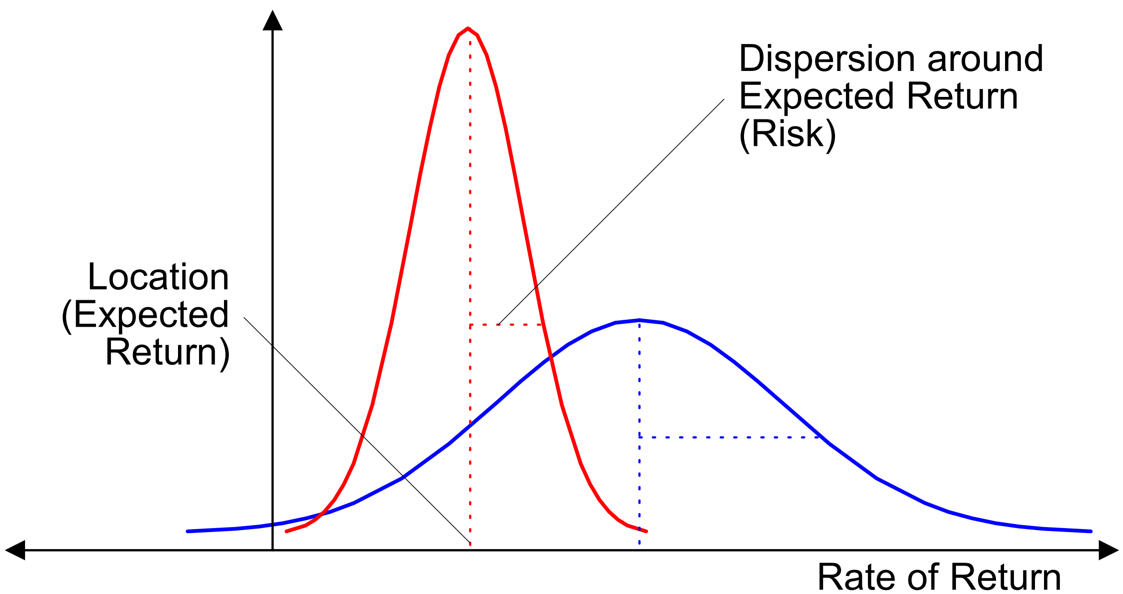


Distributions of Returns

Probability Distribution of Returns

Definition: list of all possible outcomes, together with their probabilities

Many outcomes are possible ⇒ simplify



Summary Measures

Center of Distribution: Expected Return

Dispersion of Distribution: Standard Deviation (Risk)

Expected Return

Concept
Measures center of distribution
The "typical" outcome, what is expected to happen

Calculation
Average of all possible returns, weighted by their probabilities (more likely outcomes get more weight)

$$E(k_i) = \hat{k}_i = \sum_{s=1}^{n} P_s k_{is}$$
$$= P_1 k_{i1} + P_2 k_{i2} + \cdots + P_n k_{in}$$

where $E(k_i) = \hat{k}_i =$ the expected return
P_s = probability of state *s* occurring
k_{is} = return on asset *i* if state *s* occurs

Note
The expected return is in same units as returns: percent per year.

Assets in Isolation: Stand-Alone Risk

Standard Deviation
Expected deviation of return around its expected value (i.e., expected forecast error)
Average of all possible (squared) deviations, weighted by their probabilities (more likely outcomes get more weight)
Units: same as return and expected return: percent per year

$$\sigma_i = \sqrt{\sum_{s=1}^{n} \mathrm{P}_s \left(k_{is} - \hat{k}_i\right)^2}$$
$$= \sqrt{\mathrm{P}_1\left(k_{i1} - \hat{k}_i\right)^2 + \mathrm{P}_2\left(k_{i2} - \hat{k}_i\right)^2 + \cdots + \mathrm{P}_n\left(k_{in} - \hat{k}_i\right)^2}$$

Coefficient of Variation
Allows comparison of assets whose risk and return both differ
Amount of risk endured per unit of return earned

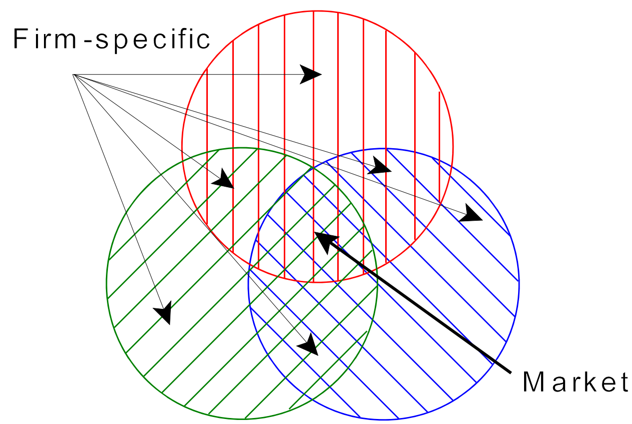
$$CV_i = \frac{\sigma_i}{\hat{k}_i}$$

Sources of Return Variability

Scope
Many events contribute to the variability of an asset's returns
Most influence *only one* (or several) asset's returns
Only a few affect *all* assets' returns

Risk of Individual Assets (when held in isolation)

$$\text{Stand-alone Risk } (\sigma_i) = \left\{ \begin{array}{c} \text{market} \\ \text{nondiversifiable} \\ \text{systematic} \end{array} \right\} \text{risk} + \left\{ \begin{array}{c} \text{firm-specific} \\ \text{diversifiable} \\ \text{unsystematic} \end{array} \right\} \text{risk}$$



Portfolios

Definition
Collection of assets

Portfolio State Return
Average of all assets' returns in a given state, weighted by their proportions in portfolio (more important assets get more weight)
Units same as returns: percent per year

$$k_{Ps} = \sum_{i=1}^{n} w_i k_{is}$$
$$= w_1 k_{1s} + w_2 k_{2s} + \cdots + w_n k_{ns}$$

where k_{Ps} = return on portfolio if state *s* occurs
w_i = proportion of portfolio invested in asset *i*: $\frac{\text{\$ invested in asset } i}{\text{\$ invested in portfolio}}$
k_{is} = return on asset *i* if state *s* occurs

Portfolio Expected Return

Expected Return
Average of all possible portfolio returns, weighted by their probabilities (more likely outcomes get more weight)
Units: percent per year

Calculation from portfolio's state-by-state returns

$$
\begin{aligned}
E\left(k_p\right) = \hat{k}_P &= \sum_{s=1}^{n} P_s k_{Ps} \\
&= P_1 k_{P1} + P_2 k_{P2} + \cdots + P_n k_{Pn}
\end{aligned}
$$

Calculation from assets' expected returns (portfolio shortcut)

$$
\begin{aligned}
E\left(k_p\right) = \hat{k}_P &= \sum_{i=1}^{n} w_i \hat{k}_i \\
&= w_1 \hat{k}_1 + w_2 \hat{k}_2 + \cdots + w_n \hat{k}_n
\end{aligned}
$$

Portfolio Risk

Standard Deviation
Expected deviation of return around its expected value (i.e., expected forecast error)
Units: percent per year

Calculation from portfolio's state-by-state returns

$$\sigma_P = \sqrt{P_1\left(k_{P1} - \hat{k}_P\right)^2 + P_2\left(k_{P2} - \hat{k}_P\right)^2 + \cdots + P_n\left(k_{Pn} - \hat{k}_P\right)^2}$$

Calculation from assets' standard deviations (portfolio shortcut)

$$\sigma_P = \sqrt{w_i^2\sigma_i^2 + w_j^2\sigma_j^2 + 2w_iw_j\sigma_{ij}}$$
$$= \sqrt{w_i^2\sigma_i^2 + w_j^2\sigma_j^2 + 2w_iw_j\sigma_i\sigma_j r_{ij}}$$

Gain from Diversification

Concept: Something for Nothing
It is possible to reduce a portfolio's risk, without reducing its expected return.

A portfolio’s standard deviation is typically *less* than the weighted average of the assets', because firm-specific variability tends to cancel out, leaving only market-wide variability. In fact, *a portfolio may even be less risky than any of the assets it contains*.

$$\sigma_P \le \sum_{i=1}^{n} w_i \sigma_i$$

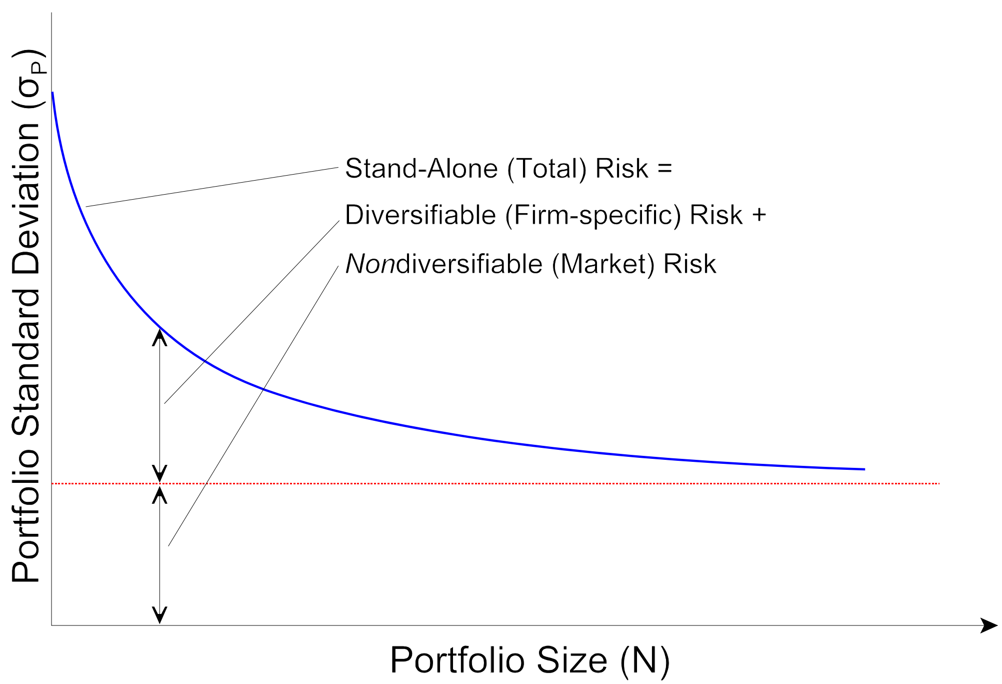
Requirement
Some diversification is possible, as long as the returns of all assets in the portfolio *do not move in lockstep*, so that firm-specific effects may cancel out

Determinants
Gain is greater the
- larger the portfolio (the more assets it contains)
- lower the correlations between the assets' returns (the weaker their interactions)

Portfolio Size and Risk

Safety in Numbers?
The larger the portfolio, the greater diversification (other things equal).
However, it is not possible to diversify away *all* risk.
Market risk will always remain, even in a well-diversified portfolio.

(Definition: a *well-diversified* or *efficient* portfolio has *only* market risk)

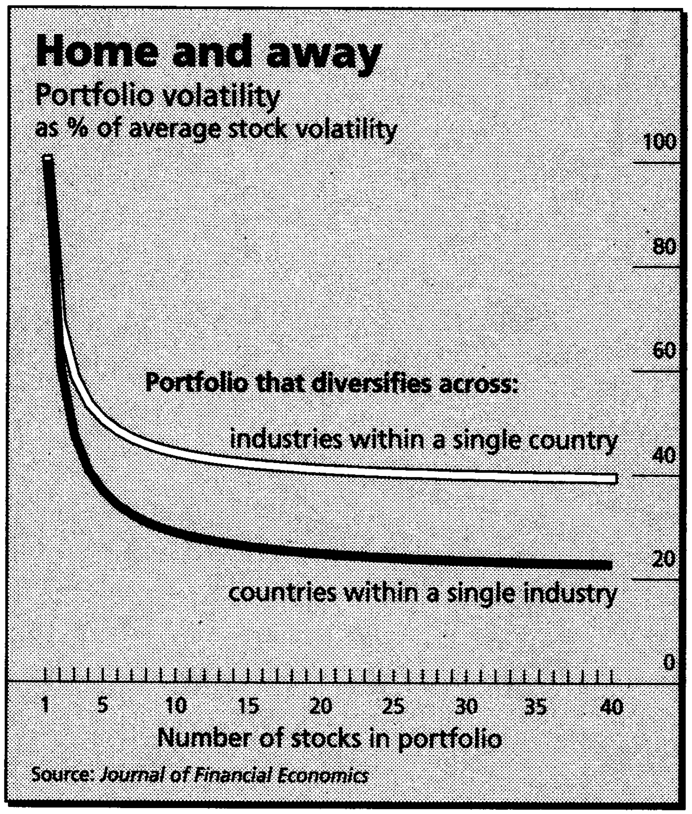


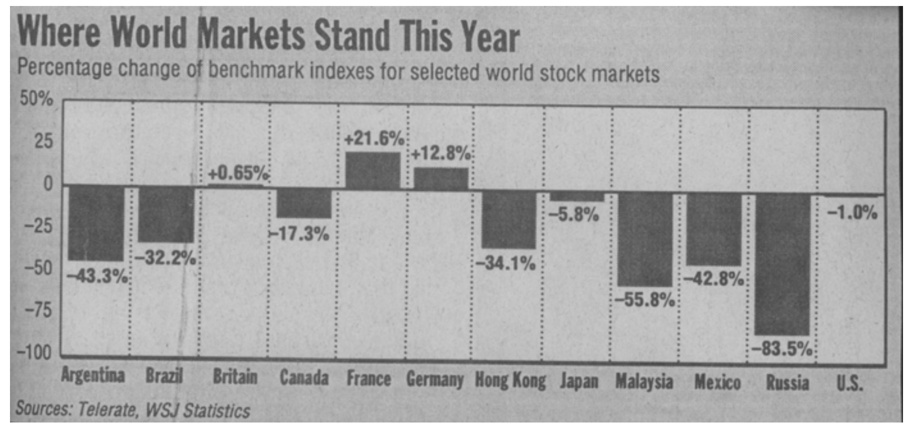
Portfolio Size: International Portfolios

Involves both Size and Correlations

When include rest of world, can form

- larger portfolios
- with lower correlations between assets (because world's markets are "out of sync")





Interactions between Returns: Covariance

Importance
"Don't put all your eggs in one basket."
To diversify, build portfolio with assets whose returns tend *not* to rise or fall together.

Covariance
Question: Do assets' returns tend to move together?

$$\sigma_{ij} = \sum_{s=1}^{n} P_s \left(k_{is} - \hat{k}_i\right)\left(k_{js} - \hat{k}_j\right)$$

$$= P_1\left(k_{i1} - \hat{k}_i\right)\left(k_{j1} - \hat{k}_j\right) + P_2\left(k_{i2} - \hat{k}_i\right)\left(k_{j2} - \hat{k}_j\right) + \cdots + P_n\left(k_{in} - \hat{k}_i\right)\left(k_{jn} - \hat{k}_j\right)$$

Interpretation
Sign tells how returns co-vary:

positive	tend to rise and fall together
negative	tend to move opposite to each other

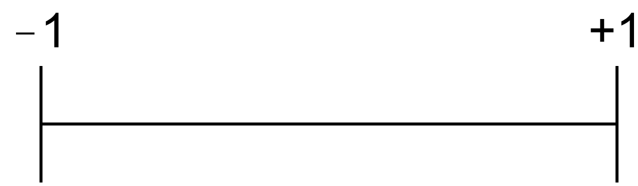
Size not meaningful

Interactions between Returns: Correlation

Correlation Coefficient
Question: How *closely* do assets' move together?

$$r_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

A scaled covariance, with a known range:

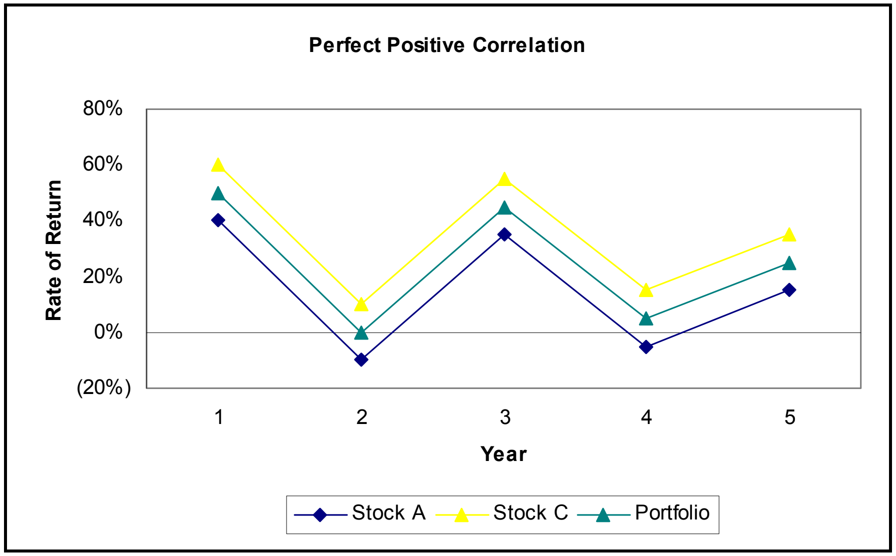


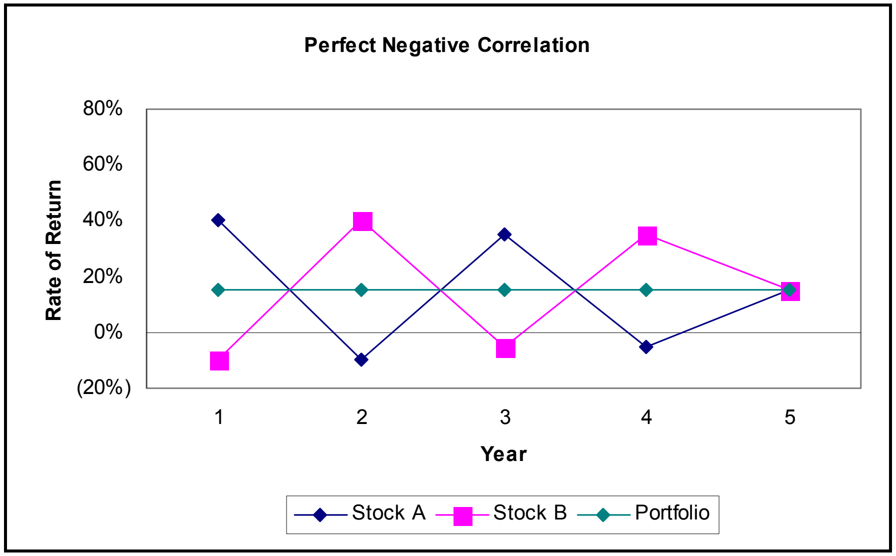
Interpretation
Sign: same as covariance (with same meaning)
Size: how closely returns co-vary

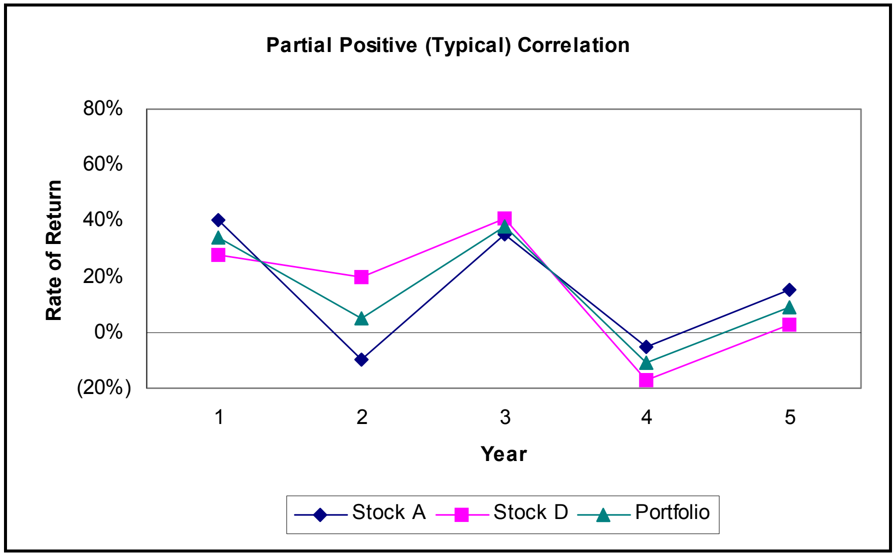
The closer r_{ij} to +1, the closer the relation between the assets' returns

The larger the correlation, the less diversification is possible

Correlation and Portfolio Risk: Illustrations







Two-Asset Portfolio: Standard Deviation

Calculation

$$\sigma_P = \sqrt{\sum_{s=1}^{n} \mathrm{P}_s \left(k_{Ps} - \hat{k}_P\right)^2}$$

$$= \sqrt{w_i^2 \sigma_i^2 + w_j^2 \sigma_j^2 + 2 w_i w_j \sigma_{ij}}$$

$$= \sqrt{w_i^2 \sigma_i^2 + w_j^2 \sigma_j^2 + 2 w_i w_j \sigma_i \sigma_j r_{ij}}$$

The smaller the correlations (r_{ij}), the smaller the portfolio risk (σ_P).

Diversification and Portfolio Efficiency

Efficient Assets

- maximize expected return (given risk)
- minimize risk (given expected return)

Only a small subset of *feasible* assets are *efficient* (most are portfolios)
Rational investors want to own only efficient assets

Risk of Individual Asset

$$\text{Total (Stand-alone) Risk}\left(\sigma_i\right) \;=\; \text{Market Risk} + \text{Firm-specific Risk}$$

Risk of Efficient (Well-Diversified) Portfolio

$$\text{Total Risk } (\sigma_P) = \left\{ \begin{array}{c} \text{market} \\ \text{nondiversifiable} \\ \text{systematic} \end{array} \right\} \text{risk}$$

Sensitive only to *general economic events*
Highly correlated with market as a whole

Assets in Portfolios: Market Risk

Market Efficiency
Since firm-specific risk is easily diversified away, no one will pay to avoid it or pay you to bear it: *only market (non-diversifiable) risk earns a risk premium*.

Asset's Market Risk
Contribution to risk of well-diversified portfolio, reflects asset's correlation with market

Beta Measures Market Risk
Beta is a scaled Covariance (or Correlation):

An asset's risk *shared with market* (σ_{iM}),

relative to market (σ_M^2 or σ_{MM})

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{\sigma_{iM}}{\sigma_{MM}} \quad \text{or} \quad r_{iM}\left(\frac{\sigma_i}{\sigma_M}\right)$$

The market (the average risky asset) is the benchmark, the standard unit of risk (β_M=1.0)

Interpreting Beta

Sign: Direction

Same meaning as sign of covariance and correlation

positive	asset's return tends to rise and fall together with market's
negative	asset's return tends to move opposite to market's (rare)

Size: Relative Volatility

How much asset's returns tend to change when market's return changes by 1%

$$\beta_i = \frac{\%\Delta k_i}{\%\Delta k_M}$$

Ratio: Measures asset's market risk compared to market's own risk

$$\beta_i \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1.0 \Leftrightarrow \text{asset } i \text{ is} \begin{Bmatrix} \text{riskier than} \\ \text{as risky as} \\ \text{less risky than} \end{Bmatrix} \text{ the market}$$

If asset's beta = 1.4, it is 1.4 times as risky as the market (it has 40% more market risk than the average risky asset)

Other Useful Properties of Beta

Forecast
For given change in market return, can forecast change in asset's return:

$$\%\Delta k_i = \beta_i \left(\%\Delta k_M \right)$$

Portfolio Beta
Much simpler than portfolio standard deviation: a simple weighted average of the assets' betas, like portfolio returns

$$\beta_P = \sum_{i=1}^{n} w_i \beta_i$$

Pricing Risk: Individual Assets

Risk Premium

Extra return required to bear a given amount of risk

$$RP_i = \begin{pmatrix} \text{Quantity} \\ \text{of} \\ \text{Risk} \end{pmatrix}_i \times \begin{pmatrix} \text{Price} \\ \text{of} \\ \text{Risk} \end{pmatrix}$$

Since asset *i*'s risk is measured relative to the market (β_i), so is its risk premium (RP_i):

$$\begin{aligned} RP_i &= \beta_i \left(RP_M \right) \\ &= \beta_i \left(k_M - k_{RF} \right) \end{aligned}$$

Price of Market Risk

The market risk premium, $k_M - k_{RF}$, is the extra return (over the risk-free rate) for bearing of one unit of market risk

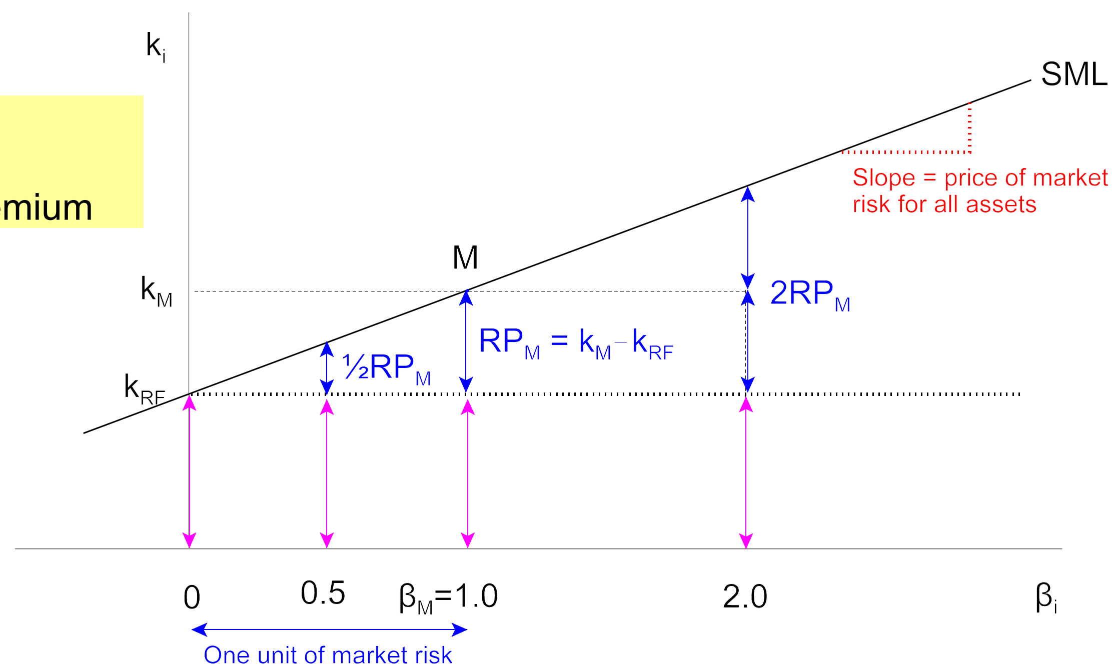
Since beta = 1.4 means an asset has 1.4 times as much market risk as the market, its risk premium will be 1.4 times the market's.

Capital Asset Pricing Model (CAPM)

Security Market Line (SML)
Required Return for *any* asset

$$k_i = k_{RF} + RP_i$$
$$= k_{RF} + \beta_i \left(k_M - k_{RF}\right)$$

All assets pay the risk-free rate.
Risky assets also pay a risk premium.
The riskier the asset, the larger the premium

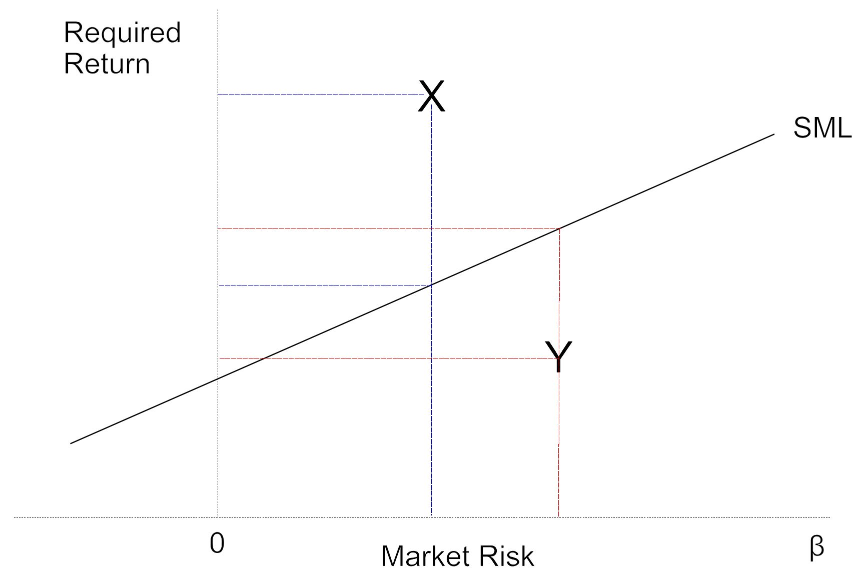


CAPM: SML and Equilibrium

Equilibrium
Expected return equals required return
Price equals value

Dynamics
Suppose an asset were to drift off the SML

$\hat{k} \sim k$?	Buy/Sell	Price	$\hat{k}$	Price Was?
>	Buy	rise	fall	Under-
=				Correct
<	Sell	fall	rise	Over-

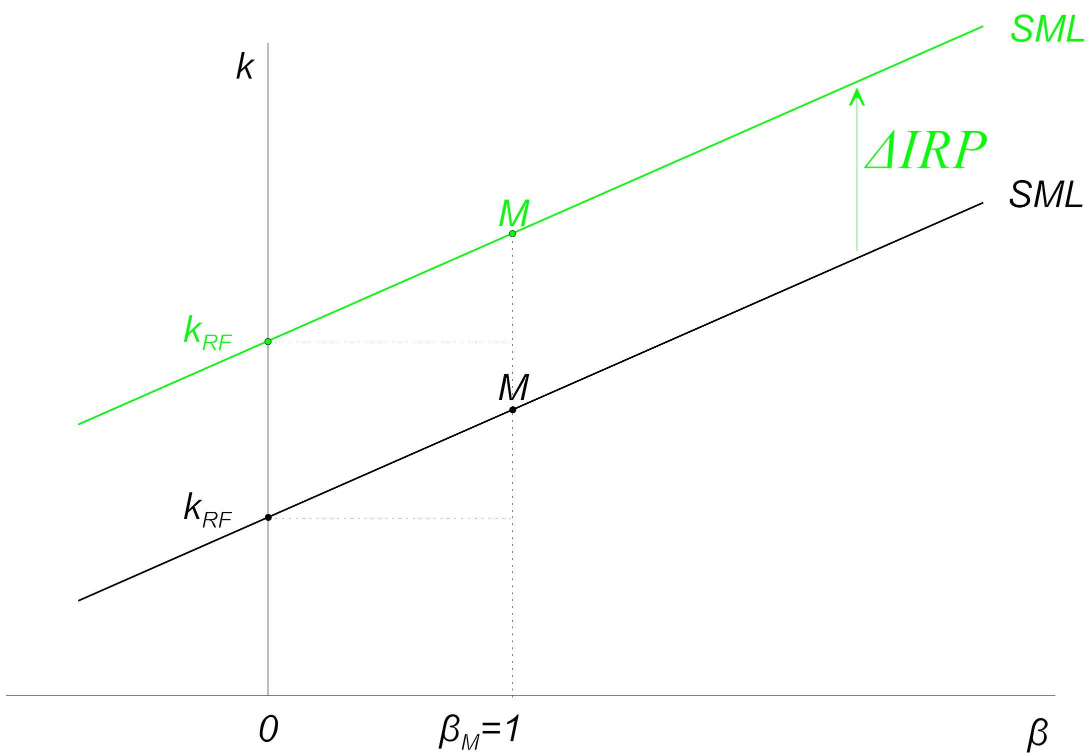


CAPM: SML and Expected Inflation

Change in Expected Inflation Rate
SML shifts in same direction

$$k_{RF} = k^* + IRP$$

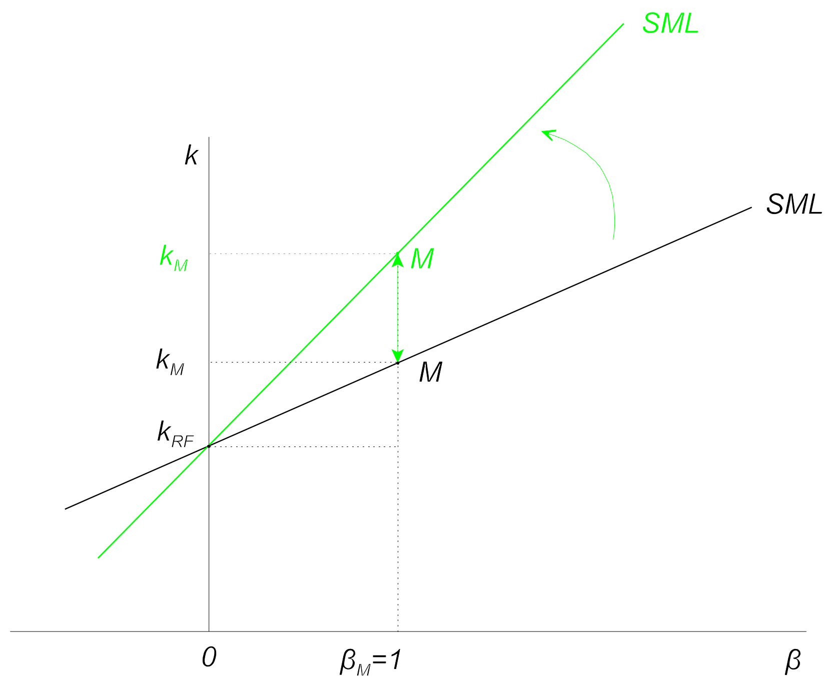
$$k_i = k_{RF} + \beta_i \left(k_M - k_{RF} \right)$$



CAPM: SML and Risk Aversion

Change in Risk Aversion
SML's slope changes in same direction

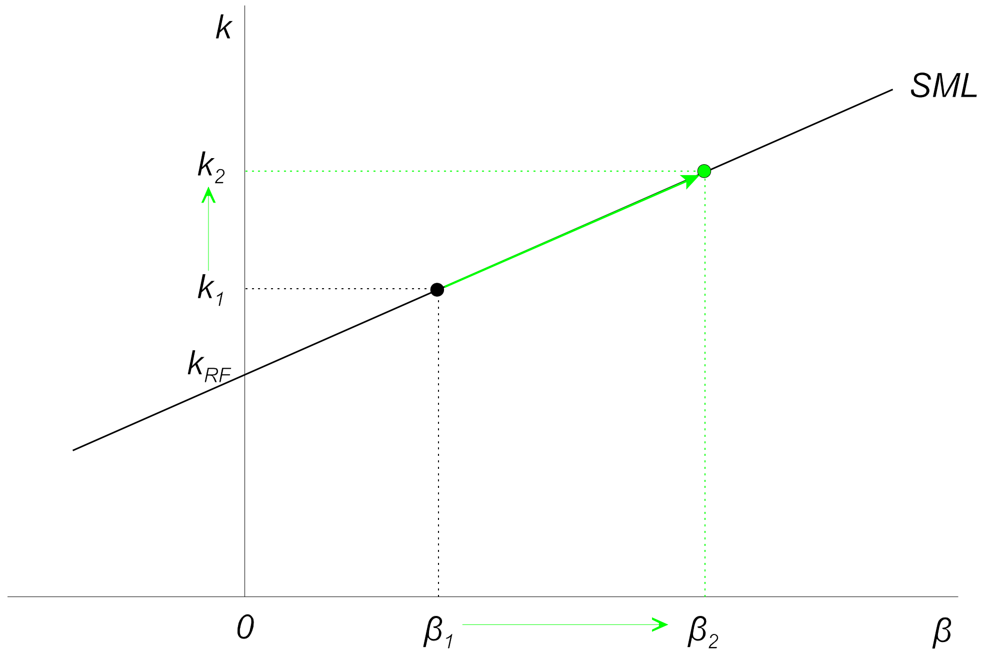
$$k_i = k_{RF} + \beta_i \left(k_M - k_{RF} \right)$$



CAPM: SML and Firm Risk

Change in Firm's Market Risk
SML unchanged

$$k_i = k_{RF} + \beta_i \left(k_M - k_{RF} \right)$$



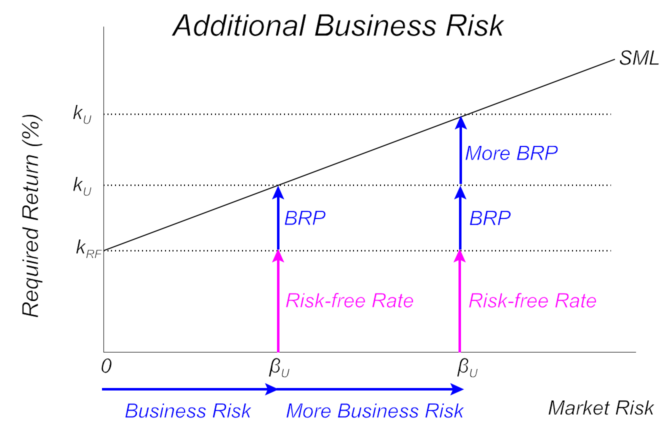
What Might Change a Firm's Market Risk (β)?
Investment Decisions (Change in Business Risk)
Financing Decisions (Change in Financial Risk)

What Might Change a Firm's Market Risk (β)?

$$\text{Market Risk}\left(\beta_i\right) = \text{Business Risk}_\text{i} + \text{Financial Risk}_\text{i}$$

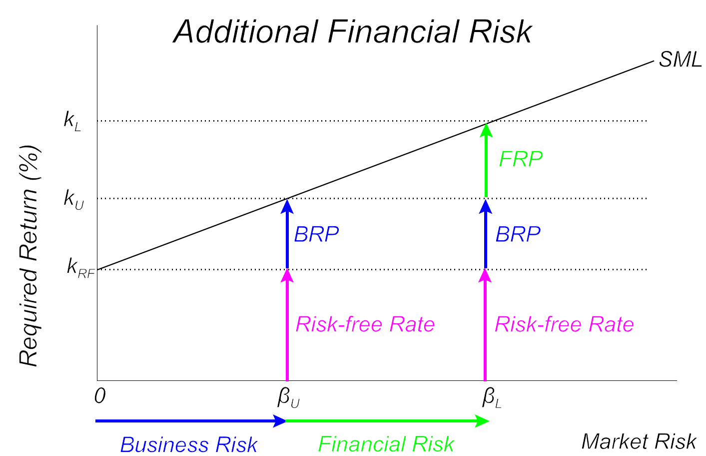
Business Risk: Investment Decisions
Determined on asset side of balance sheet by
Industry
Production Technique

Measured by unlevered (all-equity) or asset beta



Financial Risk: Financing Decisions
Determined on claims side of balance sheet by
Financial Leverage, i.e., *fixed-cost* financing

Measured by difference between *actual* beta and unlevered (all-equity) or asset beta



Estimating Beta

From Probability Distribution

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{\sigma_{iM}}{\sigma_{MM}} = r_{iM}\left(\frac{\sigma_i}{\sigma_M}\right)$$

where

$$\sigma_{iM} = \sum_{j=1}^{n} w_j \sigma_{ij} = w_1\sigma_{i1} + w_2\sigma_{i2} + \cdots + w_n\sigma_{in}$$

$$r_{iM} = \sum_{j=1}^{n} w_j r_{ij}$$

From Historical Data

Regress asset's returns on market's returns

Slope of fitted line: market risk (β)

Scatter around line: firm-specific risk (R^2)

Estimating Beta: Linear Regression

Data

Year	Market	Stock J
1	23.8%	38.6%
2	-7.2%	-24.7%
3	6.6%	12.3%
4	20.5%	8.2%
5	30.6%	40.1%

Basic Statistics

	Market	Stock J
$\bar{k}$	14.9%	14.9%
σ_k	15.1%	26.5%
r_{MJ}		0.91

Regression Estimates

Intercept	-0.09
Slope	1.60
R^2	0.83