Time Value of Money

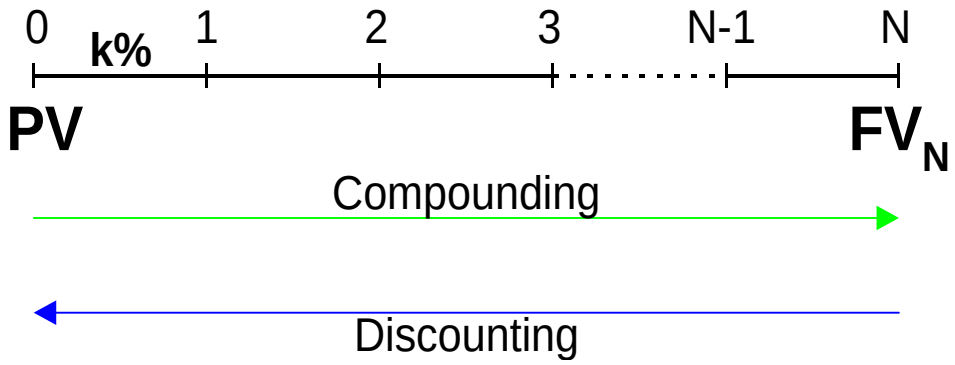
- Issue
  - A dollar today is worth more than a dollar tomorrow ("Time Preference")
    - Why?
    - How much more?
- Importance
  - Determines asset value, e.g., for common stock
    - Size
    - *Timing*
    - Risk

Cash Flow Patterns

- Single Cash Flow
  - lump sum (PV, FV)
- Annuity
  - level, regular stream (PMT)
- Perpetuity
  - perpetual annuity (PMT)
- Uneven Stream
  - CF_t

Tools: Time Lines

- Cash Flows'
  - Size
  - Direction
  - Timing
- Knowns
- Unknowns



Timelines: Practice

- Sketch timelines illustrating
  - $100 to be received at the end of Year 2
  - a 3-year ordinary annuity of $100
  - $50 paid now, $100 received in 1 year, $75 received in 2 years, and $50 received in 3 years

Tools: Calculator

Variables in Problem

- **N**umber of Periods
- **I**nterest per Period
- **P**resent **V**alue (at t = 0)
- Size of Annuity (**PMT**)
- **F**uture **V**alue (at t = N)

Note: Assign values to variables from timeline: first *number*, then **key**

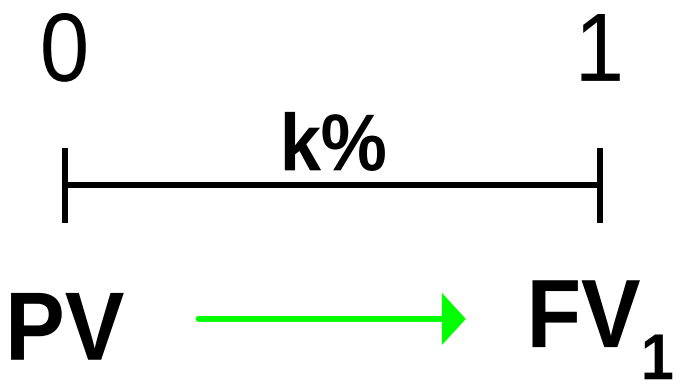
Notes:
Set **P/YR** to 1
Set **BEG/END** to END

Single CFs: Characteristics

- Future Value (FV)
  - How much will something grow to be worth?
- Present Value (PV)
  - How much is some future CF worth now?
- Rate of Return or Yield (k, r or i)
  - How fast is something earning/growing?
- Number of Periods (N)
  - How long to reach target?

Single CF: FV (1 period)

- Bank offers CD
  - Deposit
  - Maturity
  - Rate
- What's its payoff?



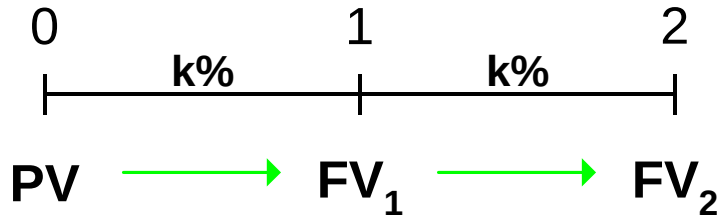
$$\text{Payoff} = \text{Principal} + \text{Interest}$$

$$FV_1 = PV + k(PV)$$

$$= PV(1+k)$$

Single CF: FV (2 periods)

- If roll over, what's payoff in 2 years?



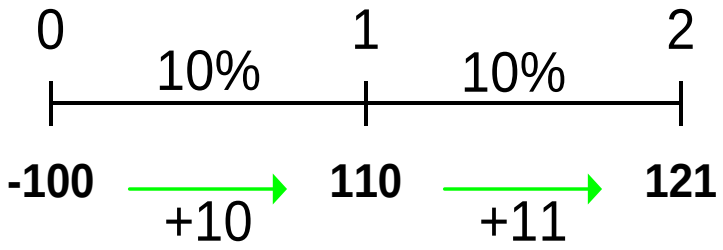
$$\text{Payoff} = \text{Principal} + \text{Interest}$$

$$FV_2 = FV_1(1+k)$$

$$= PV(1+k)^2$$

Single CF: FV (Example)

- Find FV of $100 deposit earning 10% for
  - 1 year
  - 2 years



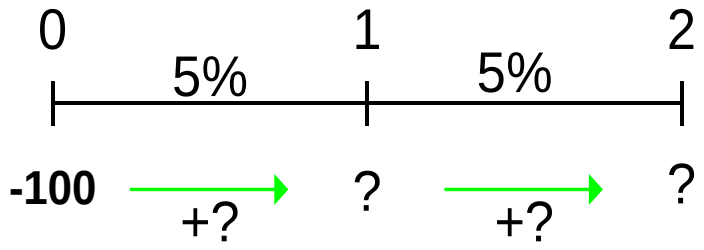
Note: interest “compounds”

Inputs	1	10	-100		
	N	**I/YR**	**PV**	**PMT**	**FV**
Output					110.00

Inputs	2	10	-100		
	N	**I/YR**	**PV**	**PMT**	**FV**
Output					121.00

Single CF: FV (Practice)

- Find FV of $100 deposit earning 5% for
  - 1 year
  - 2 years



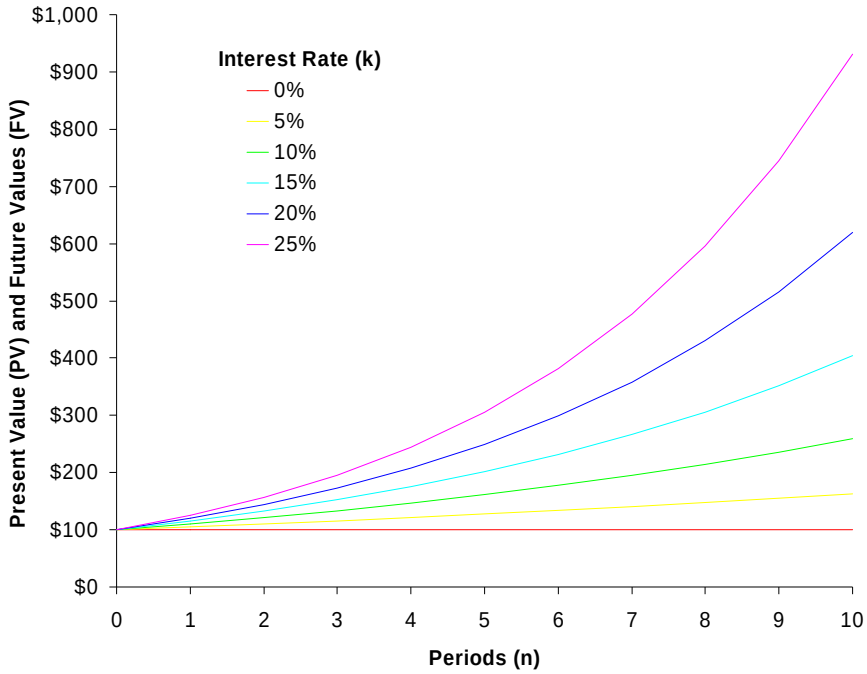
Inputs	1	5	-100		
	N	**I/YR**	**PV**	**PMT**	**FV**
Output					?

Inputs	2	5	-100		
	N	**I/YR**	**PV**	**PMT**	**FV**
Output					?

Single CF: FV (Relationships)

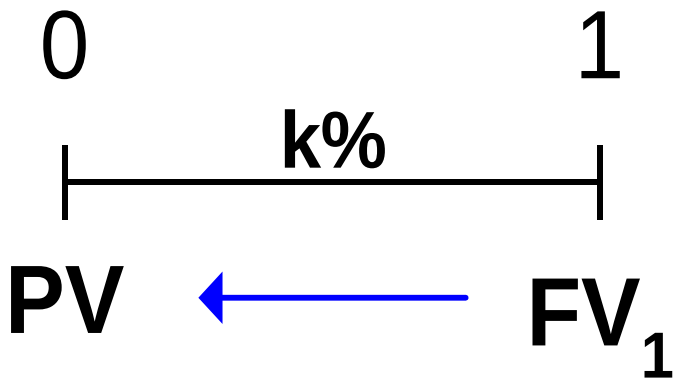
- You are indifferent between PV and FV
- Given PV, FV is *larger*
  - the larger k
  - the larger n

$$FV_n = PV(1+k)^n$$



Single CF: PV

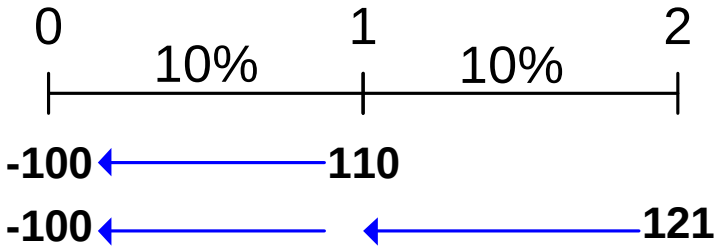
- Investment promises FV_1 in 1 year
- If pass, you can earn k% (opportunity cost)
- What's it worth *now*?
  - What's the *most* you should pay for it?



$$PV = \frac{FV_1}{1+k}$$

Single CF: PV (Example)

- If opportunity cost is 10%, find PV of
  - $110 received in 1 year
  - $121 received in 2 years

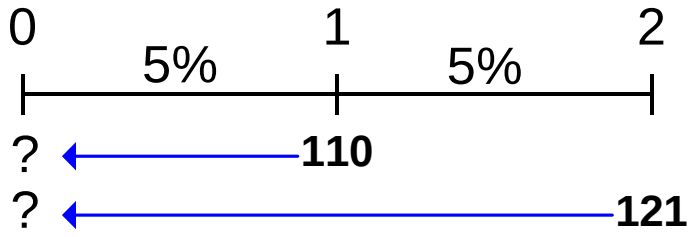


Inputs	1	10			110
	N	**I/YR**	**PV**	**PMT**	**FV**
Output			-100.00		

Inputs	2	10			121
	N	**I/YR**	**PV**	**PMT**	**FV**
Output			-100.00		

Single CF: PV (Practice)

- If opportunity cost is 5%, find PV of
  - $110 received in 1 year
  - $121 received in 2 years



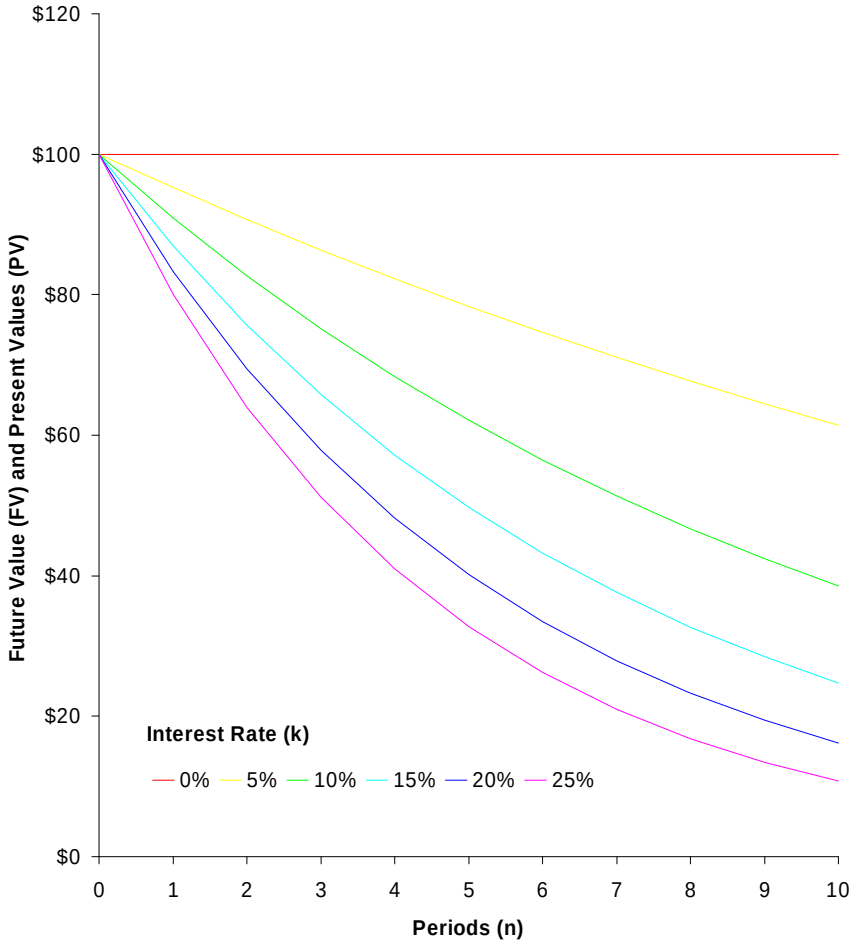
Inputs	1	5			110
	N	**I/YR**	**PV**	**PMT**	**FV**
Output			?		

Inputs	2	5			121
	N	**I/YR**	**PV**	**PMT**	**FV**
Output			?		

Single CF: PV (Relationships)

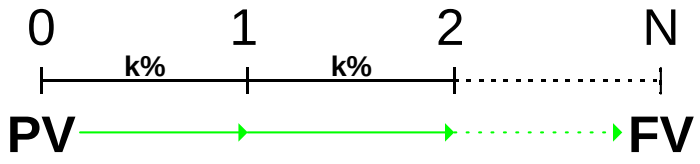
- You are indifferent between PV and FV
- Given FV, PV is *smaller*
  - the larger k
  - the larger n

$$PV = \frac{FV_n}{(1+k)^n}$$



Single CF: Rate

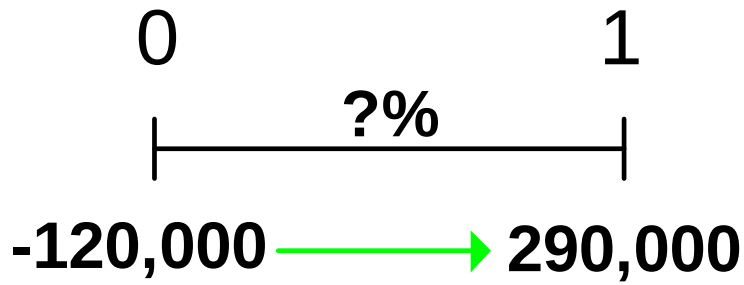
- What rate connects PV and FV over N periods?
  - Growth Rate
  - Rate of Return or Yield
- Often difficult to solve without financial calculator



$$FV_n = PV(1+k)^n$$

Growth Rate (Example)

- Last year, your firm sold $120,000.
- This year you expect to have sold $290,000.
- How fast did your sales grow during the year?

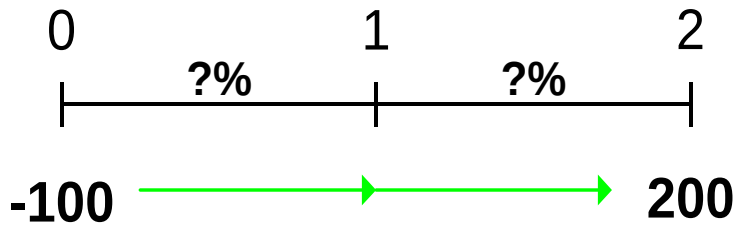


Inputs	1		-120000		290000
	N	**I/YR**	**PV**	**PMT**	**FV**
Output		141.67			

Note: sign change

Rate of Return (Practice)

- You invested $100 two years ago
- You just cashed out for $200
- What rate of return (yield) did you earn?



	N	I/YR	PV	PMT	FV
Inputs	2		-100		200
Output		?			

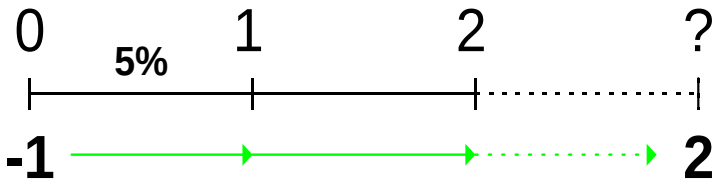
Single CF: Periods

- How long will it take to grow PV into FV, if it grows at a periodic rate of k%?

$$FV_n = PV(1+k)^n$$

Single CF: Periods (Practice)

- How long will it take to double your money, if you can earn 5% per year?



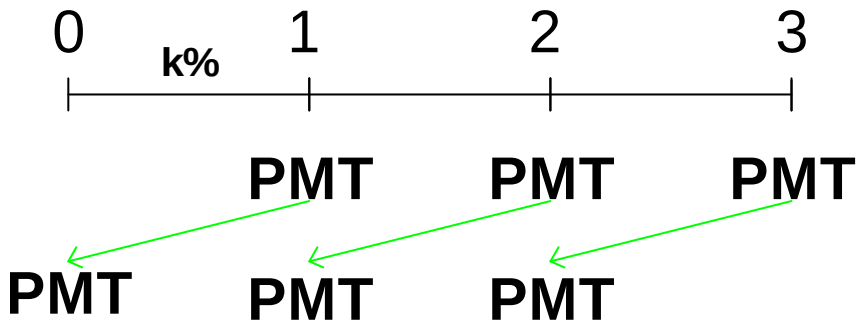
Inputs		5	-1		2
	N	**I/YR**	**PV**	**PMT**	**FV**
Output	?				

Annuities: Characteristics

- Future Value (FV)
- Present Value (PV)
- Rate of Return or Yield (k, r or i)
- Number of Periods *and Payments* (N)
- Payment (PMT)

Types of Annuities

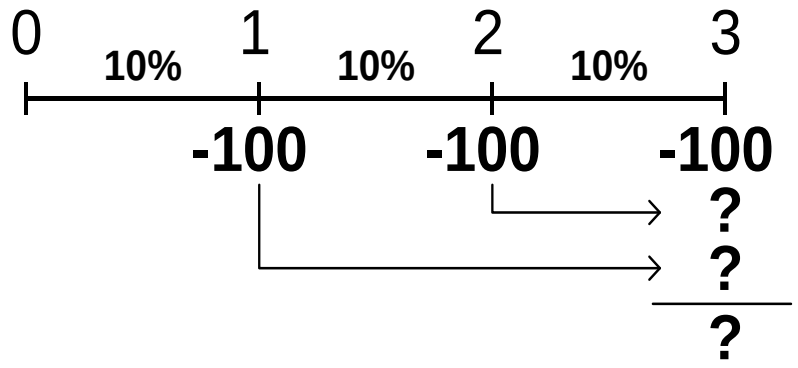
- Ordinary (deferred)
  - PMT at *end* of period
- Due (in advance)
  - PMT at *beginning* of period



Note: Set **BEG/END** key to BEG for Annuity Due, reset to END when finished

Ordinary Annuity: FV

- Deposit $100 per year for 3 years, beginning in 1 year.
- If you can earn 10%, how much will you have in 3 years?

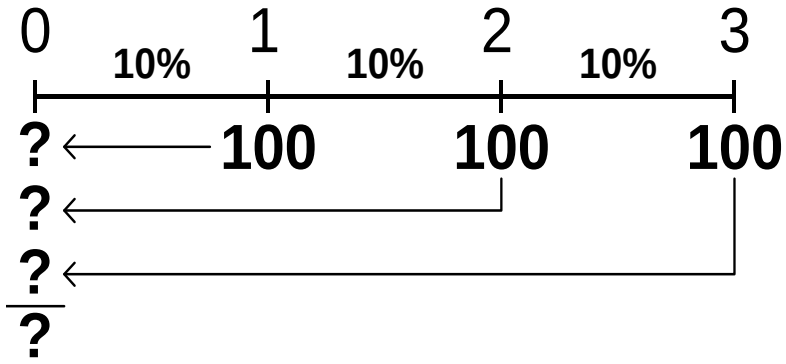


$$FVA = \sum_{t=1}^{n} PMT\,(1+k)^{n-t}$$

Inputs	3	10		-100	
	N	**I/YR**	**PV**	**PMT**	**FV**
Output					?

Ordinary Annuity: PV

- A contract will pay $100 per year for 3 years, beginning in 1 year.
- If your opportunity cost is 10%, how much is it worth?



$$PVA = \sum_{t=1}^{n} \frac{PMT}{(1+k)^t}$$

	N	I/YR	PV	PMT	FV
Inputs	3	10		100	
Output			?		

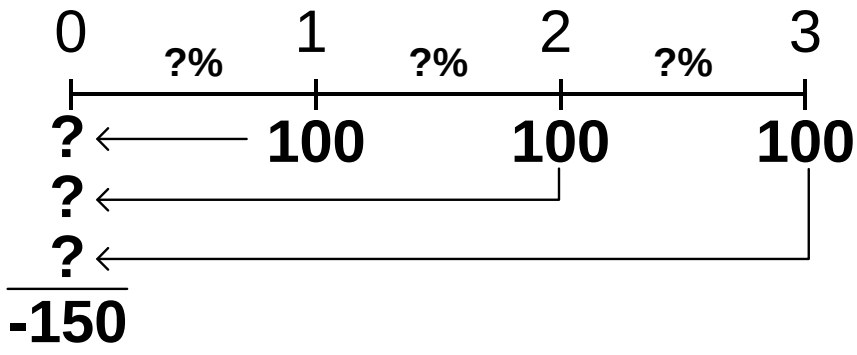
Comparing CFs, PVs and FVs

If your opportunity cost is 10%, would you rather have

- an annuity
  - $100 per year for 3 years, starting in 1 year
- its PV
  - $248.69 now
- or its FV?
  - $331 in 3 years

Ordinary Annuity: Rate

- A contract will pay $100 per year for 3 years, beginning in 1 year.
- If it costs $150, what is its rate of return?

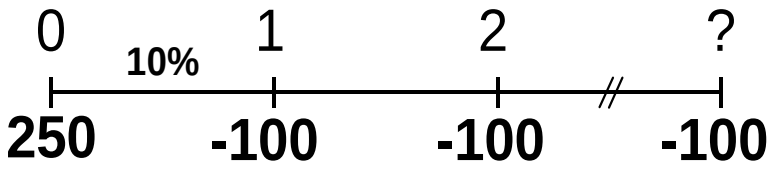


Inputs	3		-150	100	
	N	**I/YR**	**PV**	**PMT**	**FV**
Output		?			

Note: sign change

Ordinary Annuity: Periods

- You borrowed $250 at 10%.
- If you can repay $100 per year, beginning in 1 year, how long to pay it off?

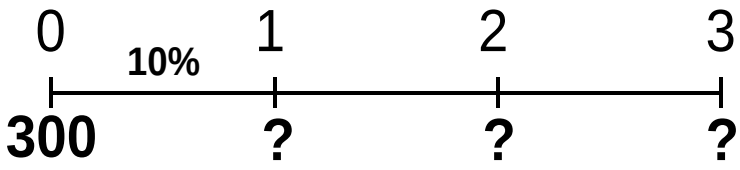


Inputs		10	250	-100	
	N	I/YR	PV	PMT	FV
Output	?				

Note: sign change

Ordinary Annuity: Payment

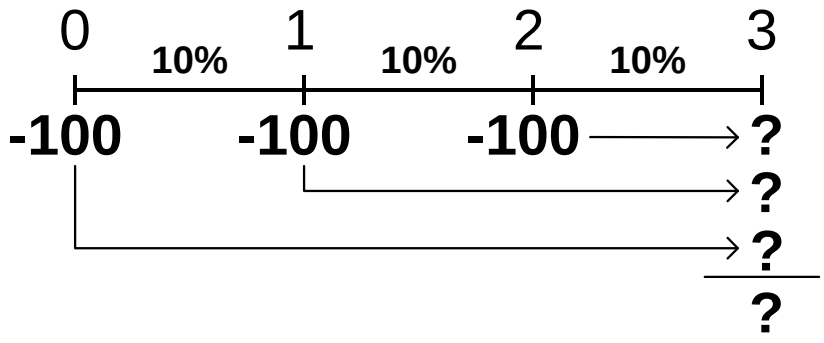
- You borrowed $300 at 10%.
- How much must you pay per year, beginning next year, in order to pay it off in exactly 3 years?



Inputs	3	10	300		
	N	**I/YR**	**PV**	**PMT**	**FV**
Output				?	

Annuity Due: FV

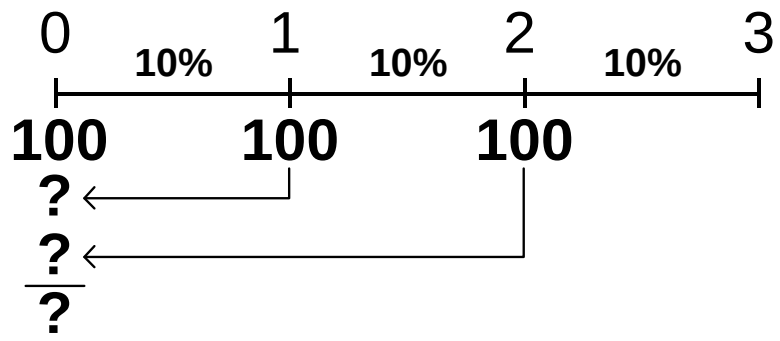
- Deposit $100 per year for 3 years, beginning now.
- If you can earn 10%, how much will you have in 3 years?
- How compares to ordinary annuity?



Inputs	3	10		-100	
	N	**I/YR**	**PV**	**PMT**	**FV**
Output					?

Annuity Due: PV

- A contract pays $100 per year for 3 years, beginning now.
- If your opportunity cost is 10%, how much is it worth?
- How compares to ordinary annuity?



Inputs	3	10		100	
	N	**I/YR**	**PV**	**PMT**	**FV**
Output			?		

Comparing Annuities' Values

- FV
  - larger for annuity due than for otherwise identical ordinary annuity by 1 period's interest

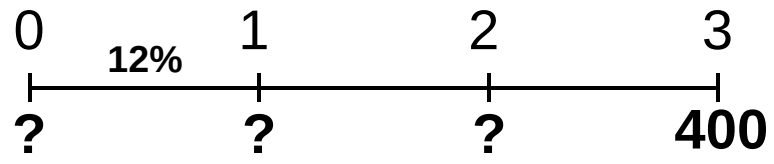
$$FVA_{due} = FVA_{ordinary}(1+k)$$

- PV
  - larger for annuity due than for otherwise identical ordinary annuity by 1 period's interest

$$PVA_{due} = PVA_{ordinary}(1+k)$$

Annuity Due: Payment

- You want to accumulate $400 within 3 years.
- You can earn 12% per year.
- How much must you invest each year, beginning now?



Inputs	3	12			400
	N	**I/YR**	**PV**	**PMT**	**FV**
Output				?	

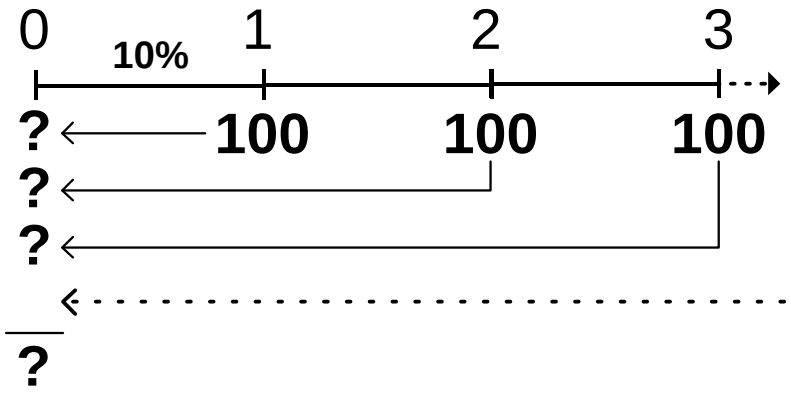
Perpetuities: Characteristics

- Present Value (PV)
- Rate of Return or Yield (k, r or i)
- Payment (PMT)

Note: Functions *not* programmed into calculator

Perpetuities: PV (Practice)

- A rich uncle has left you $100 per year forever, beginning next year.
- You can earn 10% per year.
- For how much would you sell your inheritance?

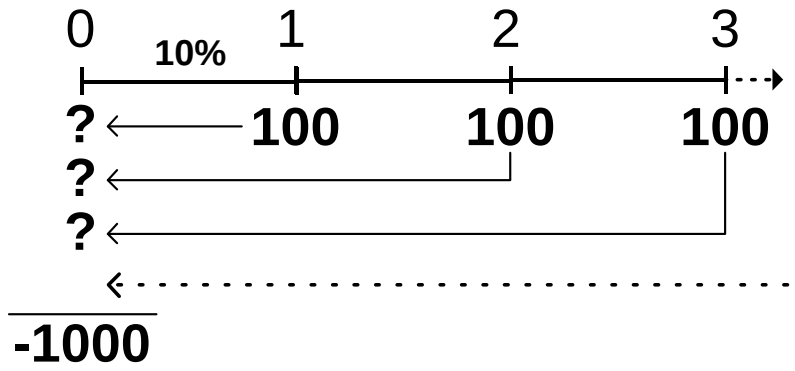


$$PV = \frac{PMT}{k}$$

Note: can approximate on calculator as annuity with *very* large N

Perpetuities: Rate

- A contract will pay $100 per year forever, beginning next year.
- If it costs $1000, what is its rate of return?



$$k = \frac{PMT}{PV}$$

Uneven Streams: Characteristics

- Future Value (FV)
- Present Value (PV)
- Rate of Return or Yield (k, r or i)
- Missing Value

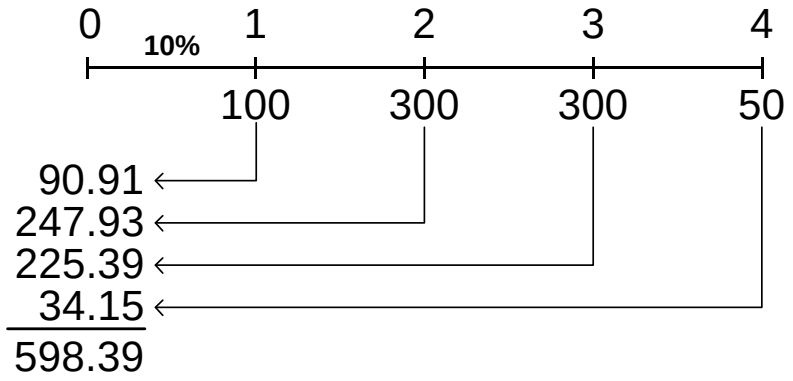
Note: Use **CF**, **NPV**, and **IRR** keys

Uneven Streams: Special Keys

- **CF**
  - *clear calculator/worksheet before starting*
  - enter CFs, with sign, *starting at Time 0*
- **NPV**
  - finds PV
  - does *not* flip sign of result
- **IRR**
  - finds rate (otherwise difficult-to-impossible)

Uneven Stream: PV (Example)

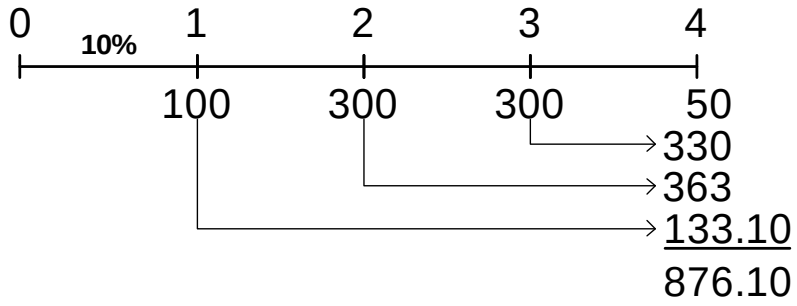
- You are offered this contract.
- Your opportunity cost is 10%.
- What's it worth?



Note: $CF_0 = 0$

Uneven Stream: FV (Example)

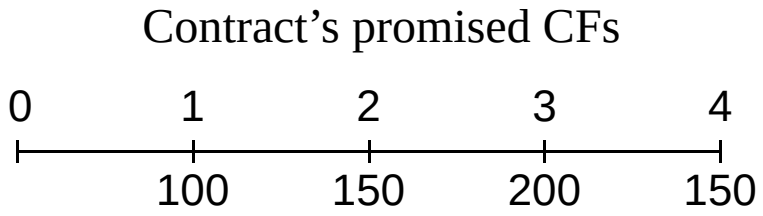
- You deposit money into an account.
- You can earn 10% per year.
- What will it grow to in 4 years?
  - Hint: Find **NPV**, then **FV**

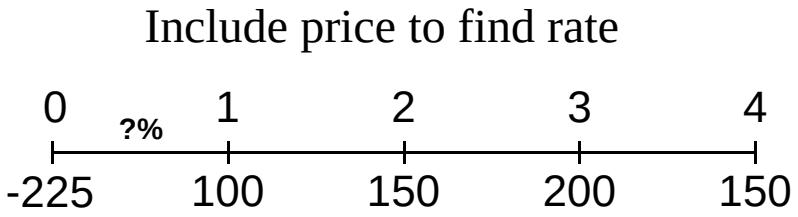


Note: $CF_0 = 0$

Uneven Stream: Rate (Example)

- Someone offers to sell you this contract for $225 today.
- What rate of return is offered?
  - Find **IRR**





Note: $CF_0 = -225$

Non-Annual Compounding

- If interest compounded more than once per year, you
  - can earn more interest on interest (FV larger)
  - will forgo more interest on interest (PV smaller)

m periods per year

- 2 semiannually
- 4 quarterly
- 12 monthly
- 360/365 daily

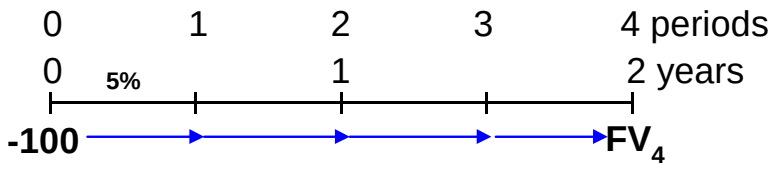
Converting Inputs

- Convert *recurring* items to periodic basis (e.g., monthly)
  - periodic rate
  - number of periods
  - payments (if any)

Conversion	for n years
Periods per Year	m
Periodic Rate	k/m
Number of Periods	mn
Periodic Payment	PMT/m

Semiannual: FV (Practice)

- You put $100 into an account paying 10% per year, compounded semiannually.
- How much will you have in 2 years?
  - Hint: Like 4-year CD with 5% annual rate.

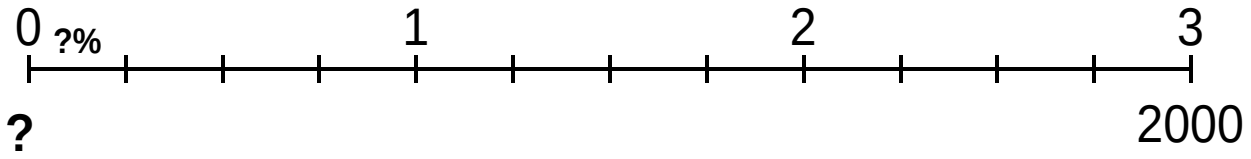


Convert	for 2 years
Periods per Year	2
Periodic Rate	5%
Number of Periods	4

Inputs	4	5	-100		
	N	I/YR	PV	PMT	FV
Output					?

Quarterly: PV (Practice)

- You are offered $2000 in 3 years.
- If you can earn 8% per year, compounded quarterly, what is it worth today?



	N	I/YR	PV	PMT	FV
Inputs	12	2			2000
Output			?		

Non-Annual: FV (Relationships)

- Given PV, k, and n, FV is *larger*
  - the larger m
    - i.e., the more frequent compounding
    - or the shorter the interval between interest dates

$$FV_{mn} = PV\left(1+\frac{k}{m}\right)^{mn}$$

Non-Annual: PV (Relationships)

- Given FV, k, and n, PV is *smaller*
  - the larger m
    - i.e., the more frequent compounding
    - or the shorter the interval between interest dates

$$PV = \frac{FV_{mn}}{\left(1+\frac{k}{m}\right)^{mn}}$$

Effective Interest Rate (EAR)

- EAR adjusts all rates to *annual* basis (i.e., as if compounded once per year)
- EAR is rate which achieves same result (PV↔FV) as nominal rate plus compounding

$$EAR = \left(1 + \frac{k}{m}\right)^{m} - 1$$

Note: For m > 1, EAR > k

EAR Calculation: Special Keys

HP calculators

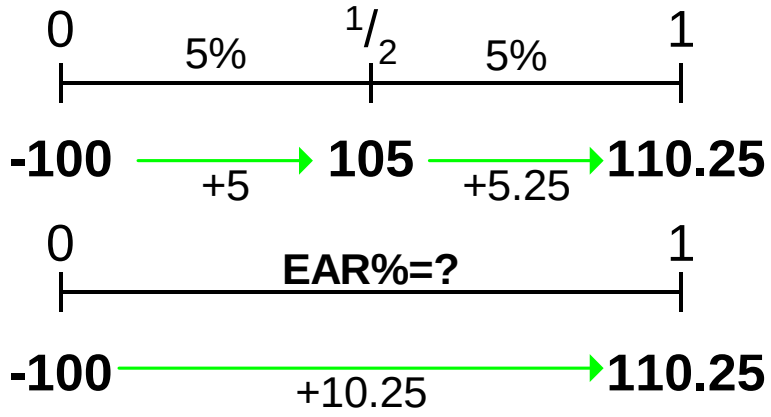
- Special keys
  - **P/YR**
    - number of compounding periods per year (m)
    - *set to 1 for most purposes*
  - **NOM%**
    - Nominal or quoted rate (k or "APR")
  - **EFF%**
    - EAR

TI calculators

- **ICONV** Worksheet (**2ND**, **2**)
  - **C/Y**
    - number of compounding periods per year (m)
  - **NOM**
    - Nominal or quoted rate (k or "APR")
  - **EFF**
    - EAR

EAR (Example)

- If you can earn 10% per year, compounded semiannually, what *effective* rate can you earn?
- 3 methods
  - cash flows
  - algebraic
  - calculator



$$EAR = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 0.1025$$

Inputs	10		2
	NOM%	**EFF%**	**P/YR**
Output		10.25	

Note: Reset **P/YR** to 1 when finished

EAR (Practice)

- Find EAR, if you earn
  - 10%, compounded annually

  - 12%, compounded quarterly

  - 5.25%, compounded daily

Types of Interest Rates

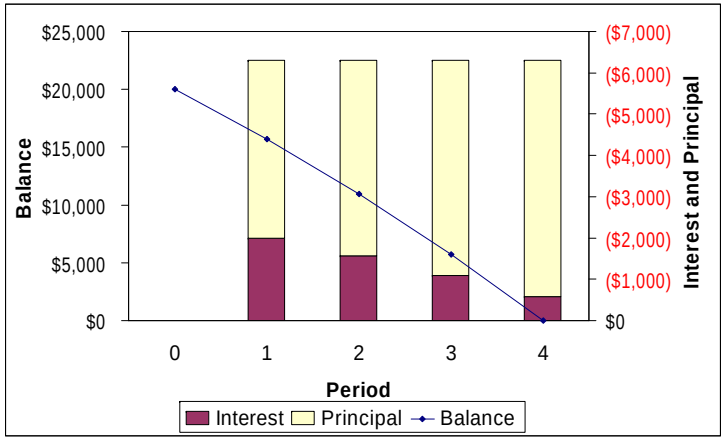
- Nominal or Quoted Rate (k, also APR)
  - stated in contracts
  - not meaningful without compounding periods
  - not used directly in calculations
- Periodic Rate (k/m)
  - actually charged/paid each period
  - generally used in calculation
  - less than nominal rate, unless $m = 1$
- Effective Rate (EAR)
  - same result as nominal rate, compounded
  - larger than nominal rate, unless $m = 1$

Amortized Loan

- Paid off with equal periodic payments, equally spaced (an annuity)
- Each payment includes both interest and principal, which change over time

Amortized Loan (Example)

- You borrow $20,000 at 10%, to be paid off over 4 years.
- How much interest and principal will you pay each year?



Period	Payment	Interest	Principal	Balance
0				$20,000.00
1	($6,309.42)	($2,000.00)	($4,309.42)	$15,690.58
2	($6,309.42)	($1,569.06)	($4,740.36)	$10,950.22
3	($6,309.42)	($1,095.02)	($5,214.40)	$5,735.82
4	($6,309.42)	($573.58)	($5,735.84)	($0.02)
	($25,237.68)	($5,237.66)	($20,000.02)	

Amortized Loan: Calculator

- Input terms of loan (PV, k, n, m, FV)
- Find PMT
- Press **AMORT** key
- Set period(s)
- To see Interest, Principal, Balance results
  - HP: press = key repeatedly
  - TI: scroll down/up