

## Risk and Return

### Issue

What is investor's *required rate of return*, i.e., the minimum rate for which he's willing to buy/hold asset?

### Importance

Discount rate for valuation, investment decisions

### Definitions

Return: annual percentage change in wealth, e.g.,  $\frac{D_1 + (P_1 - P_0)}{P_0}$  for common stock

Risk: variability of returns, deviation of actual outcome from expectation

## Investors' Attitudes

### "Rational" Investors Prefer

More Return (Greedy)

Less Risk (Risk-Averse)

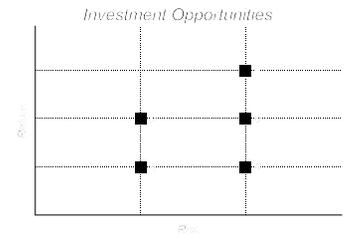
- will not bear more risk without earning more return
- will pay (or accept less return) in order to avoid risk

### Choices

Rank potential investments by comparing

- expected returns (given risks)
- risks (given expected returns)

Set of non-dominated assets is called "efficient"  
*Rational investors are interested only in efficient assets*



### Risk-Return Tradeoff

*Can't earn more return without bearing more risk*

## Required Rate of Return

### Required Rate of Return

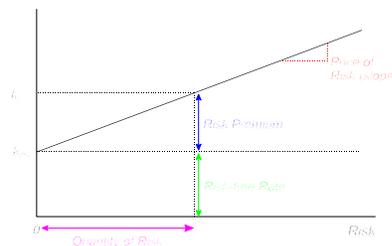
$$k_i = \text{Risk-free Rate} + \text{Risk Premium for Asset } i$$

$$= k_{RF} + RP_i$$

### Risk Premium

Extra return required to bear a given amount of risk:  $RP_i = \left( \text{Quantity of Risk} \right)_i \times \left( \text{Price of Risk} \right)$

All assets pay the risk-free rate.  
 Risky assets also pay a risk premium.  
 The riskier the asset, the larger the premium.

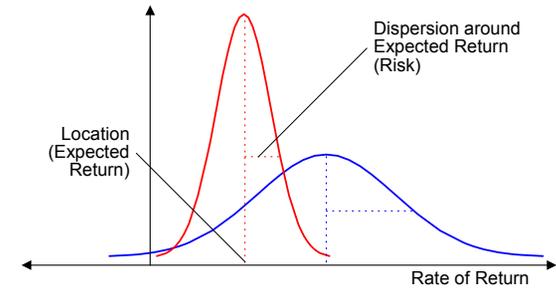


## Distributions of Returns

### Probability Distribution of Returns

Definition: list of all possible outcomes, together with their probabilities

Many outcomes are possible  $\rightarrow$  simplify



### Summary Measures

Center of Distribution: Expected Return

Dispersion of Distribution: Standard Deviation (Risk)

## Expected Return

### Concept

Measures center of distribution

The "typical" outcome, what is expected to happen

### Calculation

Average of all possible returns, weighted by their probabilities (more likely outcomes get more weight)

$$E(k_i) = \hat{k}_i = \sum_{s=1}^n P_s k_{is}$$

$$= P_1 k_{i1} + P_2 k_{i2} + \dots + P_n k_{in}$$

where  $E(k_i) = \hat{k}_i$  = the expected return

$P_s$  = probability of state  $s$  occurring

$k_{is}$  = return on asset  $i$  if state  $s$  occurs

### Note

The expected return is in same units as returns: percent per year.

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## Assets in Isolation: Stand-Alone Risk

### Standard Deviation

Expected deviation of return around its expected value (i.e., expected forecast error)  
Average of all possible (squared) deviations, weighted by their probabilities (more likely outcomes get more weight)

Units: same as return and expected return: percent per year

$$\sigma_i = \sqrt{\sum_{s=1}^n P_s (k_{is} - \hat{k}_i)^2}$$

$$= \sqrt{P_1 (k_{i1} - \hat{k}_i)^2 + P_2 (k_{i2} - \hat{k}_i)^2 + \dots + P_n (k_{in} - \hat{k}_i)^2}$$

### Coefficient of Variation

Allows comparison of assets whose risk and return both differ

Amount of risk endured per unit of return earned

$$CV_i = \frac{\sigma_i}{\hat{k}_i}$$

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## Sources of Return Variability

### Scope

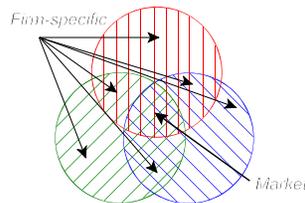
Many events contribute to the variability of an asset's returns

Most influence *only one* (or several) asset's returns

Only a few affect *all* assets' returns

### Risk of Individual Assets (when held in isolation)

$$\text{Stand-alone Risk } (\sigma_i) = \left\{ \begin{array}{l} \text{market} \\ \text{nondiversifiable} \\ \text{systematic} \end{array} \right\} \text{ risk} + \left\{ \begin{array}{l} \text{firm-specific} \\ \text{diversifiable} \\ \text{unsystematic} \end{array} \right\} \text{ risk}$$



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## Portfolios

### Definition

Collection of assets

### Portfolio State Return

Average of all assets' returns in a given state, weighted by their proportions in portfolio (more important assets get more weight)

Units same as returns: percent per year

$$k_{Ps} = \sum_{i=1}^n w_i k_{is}$$

$$= w_1 k_{1s} + w_2 k_{2s} + \dots + w_n k_{ns}$$

where  $k_{Ps}$  = return on portfolio if state  $s$  occurs

$w_i$  = proportion of portfolio invested in asset  $i$ :  $\frac{\$ \text{ invested in asset } i}{\$ \text{ invested in portfolio}}$

$k_{is}$  = return on asset  $i$  if state  $s$  occurs

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## Portfolio Expected Return

### Expected Return

Average of all possible portfolio returns, weighted by their probabilities (more likely outcomes get more weight)

Units: percent per year

### Calculation from portfolio's state-by-state returns

$$E(k_p) = \hat{k}_p = \sum_{s=1}^n P_s k_{Ps}$$

$$= P_1 k_{P1} + P_2 k_{P2} + \dots + P_n k_{Pn}$$

### Calculation from assets' expected returns (portfolio shortcut)

$$E(k_p) = \hat{k}_p = \sum_{i=1}^n w_i \hat{k}_i$$

$$= w_1 \hat{k}_1 + w_2 \hat{k}_2 + \dots + w_n \hat{k}_n$$

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## Portfolio Risk

### Standard Deviation

Expected deviation of return around its expected value (i.e., expected forecast error)

Units: percent per year

### Calculation from portfolio's state-by-state returns

$$\sigma_P = \sqrt{P_1(k_{P1} - \hat{k}_P)^2 + P_2(k_{P2} - \hat{k}_P)^2 + \dots + P_n(k_{Pn} - \hat{k}_P)^2}$$

### Calculation from assets' standard deviations (portfolio shortcut)

$$\sigma_P = \sqrt{w_i^2 \sigma_i^2 + w_j^2 \sigma_j^2 + 2w_i w_j \sigma_{ij}}$$

$$= \sqrt{w_i^2 \sigma_i^2 + w_j^2 \sigma_j^2 + 2w_i w_j \sigma_i \sigma_j r_{ij}}$$

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## Gain from Diversification

### Concept: Something for Nothing

*It is possible to reduce a portfolio's risk, without reducing its expected return.*

A portfolio's standard deviation is typically *less* than the weighted average of the assets', because firm-specific variability tends to cancel out, leaving only market-wide variability. In fact, a *portfolio may even be less risky than any of the assets it contains.*

$$\sigma_P \leq \sum_{i=1}^n w_i \sigma_i$$

### Requirement

Some diversification is possible, as long as the returns of all assets in the portfolio *do not move in lockstep*, so that firm-specific effects may cancel out

### Determinants

Gain is greater the

- larger the portfolio (the more assets it contains)
- lower the correlations between the assets' returns (the weaker their interactions)

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## Portfolio Size and Risk

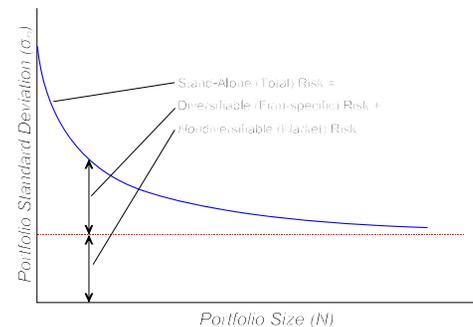
### Safety in Numbers?

The larger the portfolio, the greater diversification (other things equal).

However, it is not possible to diversify away *all* risk.

Market risk will always remain, even in a well-diversified portfolio.

(Definition: a *well-diversified* or *efficient* portfolio has *only* market risk)



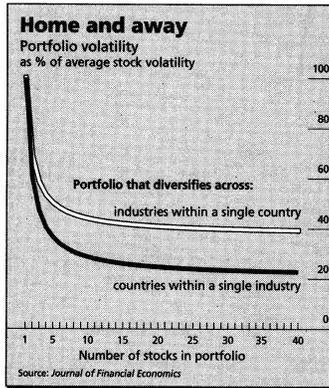
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## Portfolio Size: International Portfolios

### Involves both Size and Correlations

When include rest of world, can form

- larger portfolios
- with lower correlations between assets (because world's markets are "out of sync")



## Interactions between Returns: Covariance

### Importance

"Don't put all your eggs in one basket."

To diversify, build portfolio with assets whose returns tend *not* to rise or fall together.

### Covariance

Question: Do assets' returns tend to move together?

$$\sigma_{ij} = \sum_{s=1}^n P_s (k_{is} - \hat{k}_i)(k_{js} - \hat{k}_j)$$

$$= P_1 (k_{i1} - \hat{k}_i)(k_{j1} - \hat{k}_j) + P_2 (k_{i2} - \hat{k}_i)(k_{j2} - \hat{k}_j) + \dots + P_n (k_{in} - \hat{k}_i)(k_{jn} - \hat{k}_j)$$

### Interpretation

Sign tells how returns co-vary:

- positive tend to rise and fall together
- negative tend to move opposite to each other

Size not meaningful

## Interactions between Returns: Correlation

### Correlation Coefficient

Question: How *closely* do assets' move together?

$$r_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

A scaled covariance, with a known range:



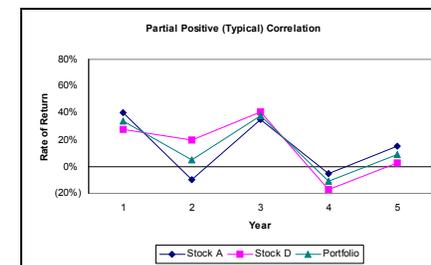
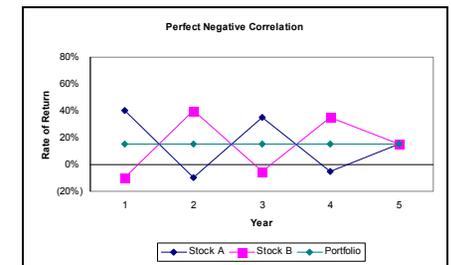
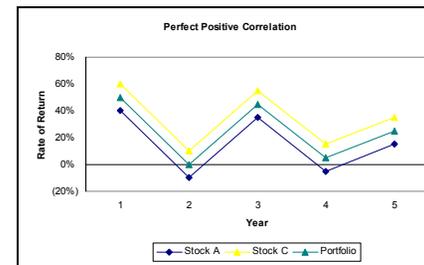
### Interpretation

Sign: same as covariance (with same meaning)

Size: how closely returns co-vary

The closer  $r_{ij}$  to +1, the closer the relation between the assets' returns  
The larger the correlation, the less diversification is possible

## Correlation and Portfolio Risk: Illustrations



## Two-Asset Portfolio: Standard Deviation

### Calculation

$$\begin{aligned}\sigma_P &= \sqrt{\sum_{s=1}^n P_s (k_{P_s} - \hat{k}_P)^2} \\ &= \sqrt{w_i^2 \sigma_i^2 + w_j^2 \sigma_j^2 + 2w_i w_j \sigma_{ij}} \\ &= \sqrt{w_i^2 \sigma_i^2 + w_j^2 \sigma_j^2 + 2w_i w_j \sigma_i \sigma_j r_{ij}}\end{aligned}$$

The smaller the correlations ( $r_{ij}$ ), the smaller the portfolio risk ( $\sigma_P$ ).

## Diversification and Portfolio Efficiency

### Efficient Assets

- maximize expected return (given risk)
- minimize risk (given expected return)

Only a small subset of *feasible* assets are *efficient* (most are portfolios)  
Rational investors want to own only efficient assets

### Risk of Individual Asset

Total (Stand-alone) Risk ( $\sigma_i$ ) = Market Risk + Firm-specific Risk

### Risk of Efficient (Well-Diversified) Portfolio

$$\text{Total Risk } (\sigma_P) = \left\{ \begin{array}{l} \text{market} \\ \text{nondiversifiable} \\ \text{systematic} \end{array} \right\} \text{ risk}$$

Sensitive only to *general economic events*  
Highly correlated with market as a whole

## Assets in Portfolios: Market Risk

### Market Efficiency

Since firm-specific risk is easily diversified away, no one will pay to avoid it or pay you to bear it: *only market (non-diversifiable) risk earns a risk premium.*

### Asset's Market Risk

Contribution to risk of well-diversified portfolio, reflects asset's correlation with market

### Beta Measures Market Risk

Beta is a scaled Covariance (or Correlation):

An asset's risk *shared with market* ( $\sigma_{iM}$ ),

*relative to market* ( $\sigma_M^2$  or  $\sigma_{MM}$ )

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{\sigma_{iM}}{\sigma_{MM}} \quad \text{or} \quad r_{iM} \left( \frac{\sigma_i}{\sigma_M} \right)$$

The market (the average risky asset) is the benchmark, the standard unit of risk ( $\beta_M=1.0$ )

## Interpreting Beta

### Sign: Direction

Same meaning as sign of covariance and correlation

- positive     asset's return tends to rise and fall together with market's
- negative     asset's return tends to move opposite to market's (rare)

### Size: Relative Volatility

How much asset's returns tend to change when market's return changes by 1%

$$\beta_i = \frac{\% \Delta k_i}{\% \Delta k_M}$$

Ratio: Measures asset's market risk compared to market's own risk

$$\beta_i \begin{cases} > \\ = \\ < \end{cases} 1.0 \Leftrightarrow \text{asset } i \text{ is } \begin{cases} \text{riskier than} \\ \text{as risky as} \\ \text{less risky than} \end{cases} \text{ the market}$$

If asset's beta = 1.4, it is 1.4 times as risky as the market (it has 40% more market risk than the average risky asset)

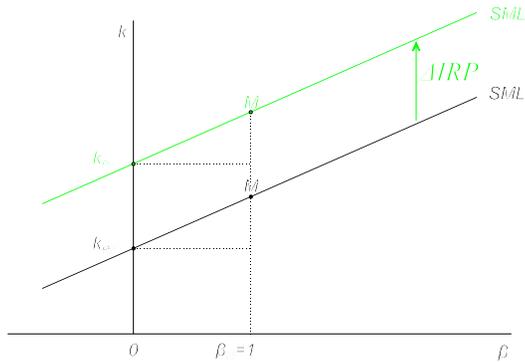


## CAPM: SML and Expected Inflation

**Change in Expected Inflation Rate**  
SML shifts in same direction

$$k_{RF} = k^* + IRP$$

$$k_i = k_{RF} + \beta_i (k_M - k_{RF})$$

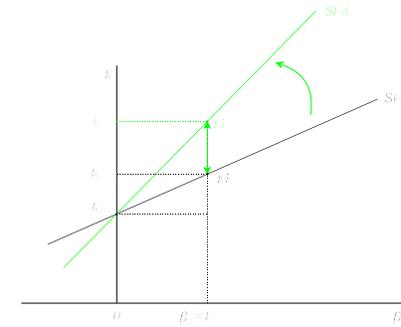


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## CAPM: SML and Risk Aversion

**Change in Risk Aversion**  
SML's slope changes in same direction

$$k_i = k_{RF} + \beta_i (k_M - k_{RF})$$

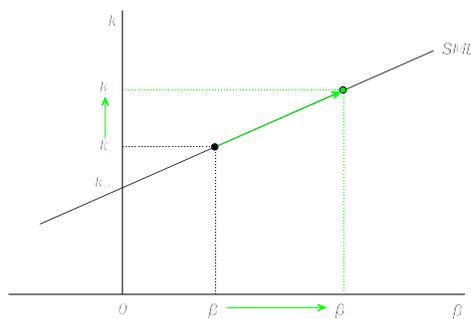


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## CAPM: SML and Firm Risk

**Change in Firm's Market Risk**  
SML unchanged

$$k_i = k_{RF} + \beta_i (k_M - k_{RF})$$



**What Might Change a Firm's Market Risk (β)?**  
Investment Decisions (Change in Business Risk)  
Financing Decisions (Change in Financial Risk)

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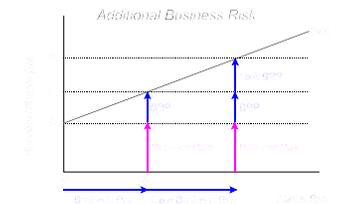
## What Might Change a Firm's Market Risk (β)?

$$\text{Market Risk } (\beta_i) = \text{Business Risk}_i + \text{Financial Risk}_i$$

### Business Risk: Investment Decisions

Determined on asset side of balance sheet by  
Industry  
Production Technique

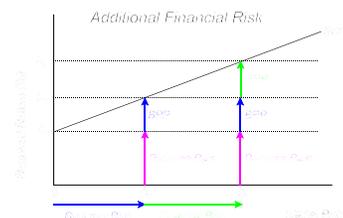
Measured by unlevered (all-equity) or asset beta



### Financial Risk: Financing Decisions

Determined on claims side of balance sheet by  
Financial Leverage, i.e., *fixed-cost* financing

Measured by difference between *actual* beta and unlevered (all-equity) or asset beta



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## Estimating Beta

### From Probability Distribution

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{\sigma_{iM}}{\sigma_{MM}} = r_{iM} \left( \frac{\sigma_i}{\sigma_M} \right)$$

where

$$\sigma_{iM} = \sum_{j=1}^n W_j \sigma_{ij} = W_1 \sigma_{i1} + W_2 \sigma_{i2} + \dots + W_n \sigma_{in}$$

$$r_{iM} = \sum_{j=1}^n W_j r_{ij}$$

### From Historical Data

Regress asset's returns on market's returns

- Slope of fitted line: market risk ( $\beta$ )
- Scatter around line: firm-specific risk ( $R^2$ )

## Estimating Beta: Linear Regression

### Data

Year	Market	Stock J
1	23.8%	38.6%
2	-7.2%	-24.7%
3	6.6%	12.3%
4	20.5%	8.2%
5	30.6%	40.1%

### Basic Statistics

	Market	Stock J
$\bar{k}$	14.9%	14.9%
$\sigma_k$	15.1%	26.5%
$r_{MJ}$		0.91

### Regression Estimates

Intercept	-0.09
Slope	1.60
$R^2$	0.83

