Bonds

Definition
Long-term Debt: legally enforceable promise by borrower to pay specified amount(s) on specified date(s)

Contractual Characteristics
Maturity (N)
Par/Face/Maturity Value (FV)
Coupon Rate, Coupon Payment (PMT)

$$\text{Coupon Payment} = \text{Coupon Rate} \times \text{Par Value}$$

Security and Seniority of claim
Call Provisions: Call Premium/Price, Date of First Call/Protection Period, Sinking Fund
Conversion Terms: Ratio, Exercise/Strike Price, Expiration
Indexing (e.g., TIPS)

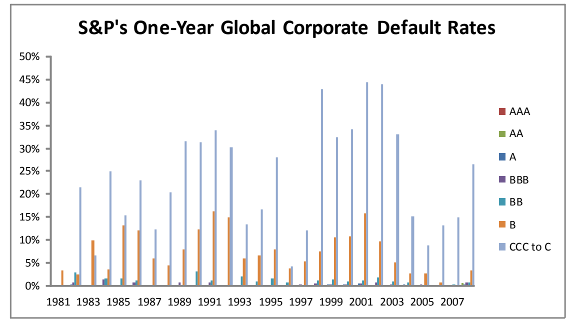
Risks
Default Risk
Interest Rate Risk (Price Risk, Risk to Wealth)
Reinvestment Rate Risk (Risk to Income)

Default Risk and Bond Ratings

Simplified Bond Ratings

	Investment Grade				Junk			
Moody's	Aaa	Aa	A	Baa	Ba	B	Caa	C
S&P	AAA	AA	A	BBB	BB	B	CCC	D

Actual Default Rates

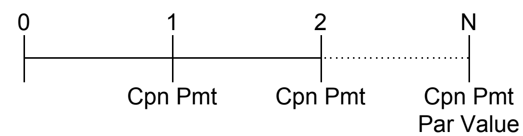


Digression: Liquidation ("Chapter 7")

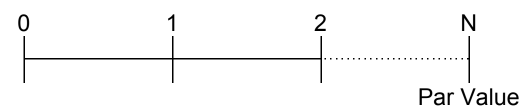
Priority of Payment
Debt owned by secured creditors (from sales of secured assets)
Trustee's costs
Wages (subject to limits)
Taxes
Unfunded pension liabilities
Debt owned by unsecured creditors (senior first, then junior)
Preferred stock
Common stock

Bonds' Cash Flow Patterns

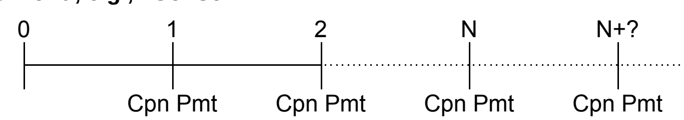
Coupon Bond



Zero-Coupon Bond, including U.S. Treasury STRIPS

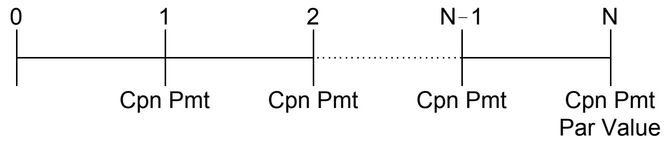


Perpetual Bond, e.g., "Consol"



Treasury STRIPS

STRIPS originate as ordinary coupon bonds



Stripping
DIY zeros: If the parts are worth more than the whole, buy bond, detach coupons and corpus, sell separately. Each separate piece becomes a new zero-coupon bond (an FV with its own maturity).

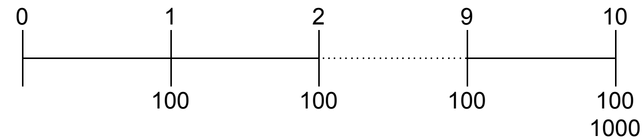
Reconstitution
If the whole is worth more than the parts, buy up parts, rebuild bond, sell.

Valuation
Because most other bonds make semiannual coupon payments (and even STRIPS start that way), value STRIPS as if they were semiannual, too.

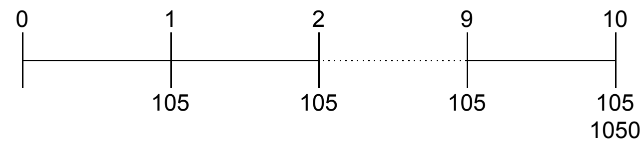
Treasury Inflation Protected Securities (TIPS)

Purpose
Discover inflation rate that *investors* expect

Cash Flows (initial for TIPS, always for ordinary Treasuries)

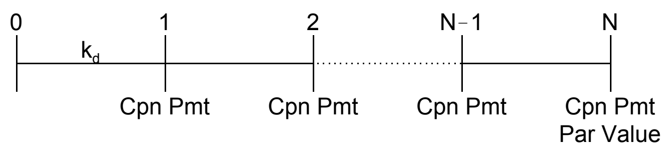


Cash Flows (Adjusted)
If the price level rises by 5 percent, par value increases by same amount:



Bond Valuation: Coupon Bond

Cash Flows



Value

$$\hat{P}_0 = \sum_{t=1}^{N} \frac{Cpn\ Pmt}{(1+k_d)^t} + \frac{Par\ Value}{(1+k_d)^N}$$

Inputs:	N	=	N
	I	=	k_d
	PMT	=	Cpn Pmt
	FV	=	Par Value
Output:	PV	=	$-\hat{P}_0$

where

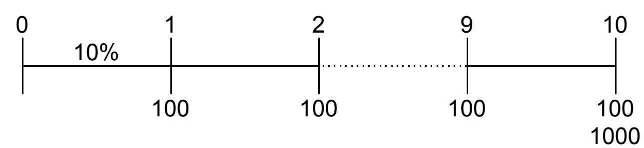
$$Cpn\ Pmt = Coupon\ Rate \times Par\ Value$$

$$k_{di} = k_{RF} + DRP_i + LP_i + MRP_i$$

Coupon Bond Valuation: Practice 1

Example
Consider a 10-year bond with a $1,000 par value and a 10% annual coupon. If the interest rate is 10%, what's it worth?

Cash Flows



Value

$$\hat{P}_0 = \sum_{t=1}^{10} \frac{100}{(1+0.10)^t} + \frac{1000}{(1+0.10)^{10}}$$

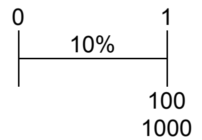
Calculator Solution

Inputs:	N	=	10
	I	=	10
	PMT	=	100
	FV	=	1000
Output:	PV	=	

Coupon Bond Valuation: Practice 2

Example
Consider a 1-year bond with a $1,000 par and a 10% annual coupon. If the interest rate is 10%, what's it worth?

Cash Flows



Value

$$\hat{P}_0 = \sum_{t=1}^{1} \frac{100}{(1+0.10)^t} + \frac{1000}{(1+0.10)^1}$$

Calculator Solution

Inputs:	N	=	1
	I	=	10
	PMT	=	100
	FV	=	1000
Output:	PV	=	

Interest Rates, Maturity and Bond Values: Practice

Suppose the Interest Rate Increases to 13%

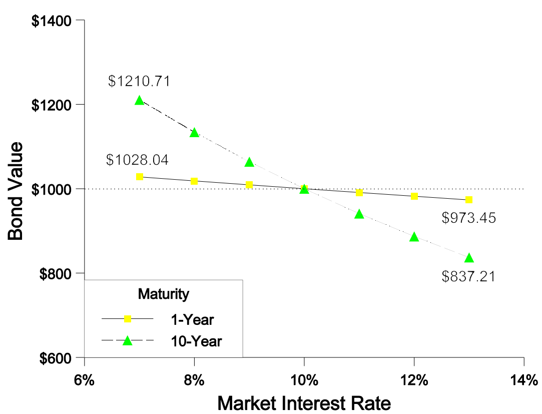
Bond			Long	Short
Inputs:	N	=	10	1
	I	=	13	13
	PMT	=	100	100
	FV	=	1000	1000
Output:	PV	=		

Suppose the Interest Rate Decreases to 7%

Bond			Long	Short
Inputs:	N	=	10	1
	I	=	7	7
	PMT	=	100	100
	FV	=	1000	1000
Output:	PV	=		

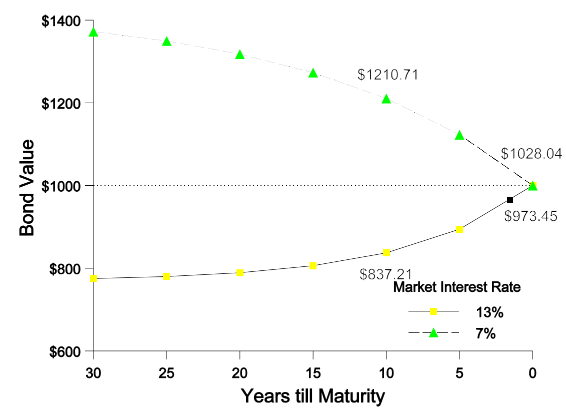
Interest Rates and Bond Value

Given Maturity



Maturity and Bond Value

Given the Interest Rate



Important Price-Yield Relationships

1. Bond values and interest rates are inversely related.

2. $\text{Market Rate} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \text{Coupon Rate} \Leftrightarrow \text{Value} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \text{Par}$

   When issuing new bonds, firms customarily set their coupon rate to equal the prevailing market interest rate, so that the new bonds will sell at par (also customarily set to 1000).

3. Interest rate risk is greater
   the longer the maturity,
   the lower the coupon rate, and
   the lower the initial yield.

   Because of interest rate risk, long-term bonds usually carry higher rates than otherwise-identical short-term bonds.

   Duration indicates the impact of all three elements simultaneously.

4. As a bond approaches maturity, its value approaches par.

Interest Rate and Reinvestment Rate Risk: Comparison

Direction of Effect
When market interest rates change:
 wealth changes in the opposite direction
 reinvested income changes in the same direction
 net effect depends on relative sizes of the two effects, which depends on maturity

Exposure Depends on Bond's Characteristics

Characteristic	Risk to Wealth	Risk to Income
Maturity: Long	High	Low
Maturity: Short	Low	High
Coupon Rate: Low	High	Low
Coupon Rate: High	Low	High

Immunization
Construct portfolios of bonds with maturities and coupon rates, so that their gain/loss due to changes in reinvestment rates offset loss/gain due to price changes. (Use bonds with duration equal to investment horizon, rebalanced periodically.)

Duration

(Macaulay) Duration
Weighted average maturity, expressed in years, of all of a bond's promised cash flows:

$$D = 1 \times \frac{PV(CF_1)}{P_0} + 2 \times \frac{PV(CF_2)}{P_0} + 3 \times \frac{PV(CF_3)}{P_0} + \cdots + N \times \frac{PV(CF_N)}{P_0}$$

$$= \frac{\sum_{t=1}^{N} t \times PV(CF_t)}{P_0} = \frac{\sum_{t=1}^{N} \frac{t \times CF_t}{(1+k_d)^t}}{P_0}$$

For any bond with a coupon rate > zero, duration < maturity. The higher the coupon rate, the shorter the duration.

Measure of price volatility
Similar to beta (β) or elasticity.
For a *small* change in market rates,

$$\%\Delta P_0 \approx -D \times \%\Delta k_d$$

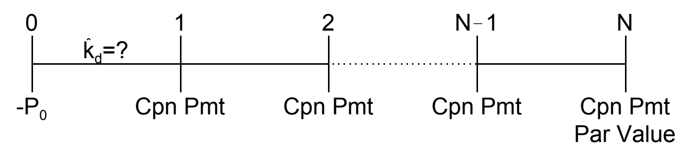
Yield to Maturity (YTM)

Definition
That discount rate which makes the PV of the promised CFs equal to the bond's price, i.e.,

$$P_0 = \sum_{t=1}^{N} \frac{Cpn\ Pmt}{(1+\hat{k}_d)^t} + \frac{Par\ Value}{(1+\hat{k}_d)^N}$$

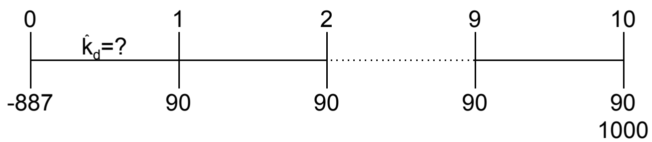
Interpretation
Promised return and, in equilibrium, *required* return or market rate; a total return

Illustration
If a bank offered the same cash flows (as the bond) for the same price (deposit), what rate of return is implied?



YTM: Practice

Discount Bond
A 9% coupon, 10-year, $1,000 par bond sells for $887. Find its YTM. Is it consistent with price-yield relationships?



Premium Bond
The same bond sells for $1,134.20. Find its YTM. Is it consistent with price-yield relationships?

Calculator Solution

Inputs:	N	=	10	10
	PV	=	-887	-1134.2
	PMT	=	90	90
	FV	=	1000	1000
Output:	I	=		

Current, Capital Gains, and Total Yields

Current Yield (CY)
Annual coupon payment, expressed as a percentage of bond's current price:

$$CY_1 = \frac{Cpn\ Pmt}{P_0} \times 100$$

Capital Gains Yield (CGY)
Annual change in bond's price (at a given interest rate), expressed as a percentage of its current price:

$$CGY_1 = \frac{\hat{P}_1 - P_0}{P_0} \times 100$$

Total Yield (TY)
Coupon payments plus capital gain, expressed as a percentage of bond's current price. In equilibrium, equal to YTM.

$$TY_1 = \frac{Cpn\ Pmt + \left(\hat{P}_1 - P_0\right)}{P_0} \times 100 = CY_1 + CGY_1$$

Current, Capital Gains, and Total Yields: Practice

Discount Bond
A 9% coupon, 10-year, $1,000 par bond sells for $887. Find its Current Yield, Capital Gains Yield and Total Yield.

Premium Bond
The same bond sells for $1,134.20. Find its Current Yield, Capital Gains Yield and Total Yield.

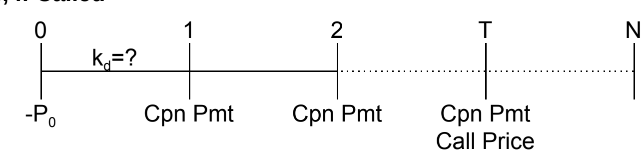
Calculator Solution

Cpn Pmt	=	90	90
P_0	=	887	1134.2
CY_1	=		
$\hat{P}_1$	=		
CGY_1	=		
TY_1	=		

Yield to First Call (YTC)

Concept
Since a bond may be called before its maturity, a bondholder can't count on earning the YTM. How much can he earn, if the bond is called as soon as it becomes eligible for call?

Cash Flows, if Called

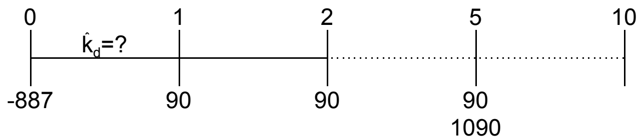


Calculation
Like calculating YTM, after substituting date of first call (T) for maturity (N) and Call Price for Par Value

$$P_0 = \sum_{t=1}^{T} \frac{Cpn\ Pmt}{(1+\hat{k}_d)^t} + \frac{Call\ Price}{(1+\hat{k}_d)^T}$$

YTC: Practice

Discount Bond
A 9% coupon, 10-year, $1,000 par bond selling for $887, is callable in 5 years at a call price of $1,090. Find its YTC.



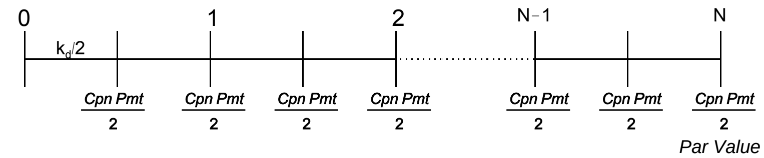
Premium Bond
The same bond sells for $1,134.20. Find its YTC.

Calculator Solution

Inputs:	N	=	5	5
	PV	=	-887	-1134.2
	PMT	=	90	90
	FV	=	1090	1090
Output:	I	=		

Semiannual Coupons

Cash Flows



Value

$$P_0 = \sum_{t=1}^{2N} \frac{\frac{Cpn\ Pmt}{2}}{\left(1+\frac{k_d}{2}\right)^t} + \frac{Par\ Value}{\left(1+\frac{k_d}{2}\right)^{2N}}$$

Calculator Solution

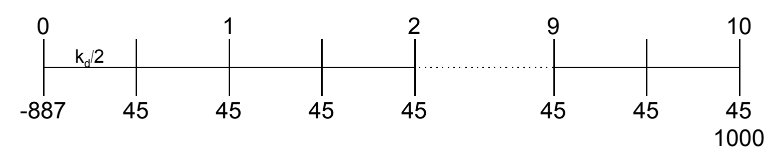
Inputs:	N	=	2N
	I	=	$k_d/2$
	PMT	=	Cpn Pmt/2
	FV	=	Par Value
Output:	PV	=	$-\hat{P}_0$

Semiannual Coupons: Value (Practice)

Interest Rate = 10%	Bond			Long	Short
	Inputs:	N	=	20	2
		I	=	5	5
		PMT	=	50	50
		FV	=	1000	1000
	Output:	PV	=		
Interest Rate = 13%	Bond			Long	Short
	Inputs:	N	=	20	2
		I	=	6.5	6.5
		PMT	=	50	50
		FV	=	1000	1000
	Output:	PV	=		
Interest Rate = 7%	Bond			Long	Short
	Inputs:	N	=	20	2
		I	=	3.5	3.5
		PMT	=	50	50
		FV	=	1000	1000
	Output:	PV	=		

Semiannual Coupons: YTM (Practice)

Discount Bond
A 9% semiannual-coupon, 10-year, $1,000 par bond sells for $887. Find its YTM. Is it consistent with price-yield relationships?



Premium Bond
The same bond sells for $1,134.20. Find its YTM. Is it consistent with price-yield relationships?

Calculator Solution

Inputs:	N	=	20	20
	PV	=	-887	-1134.2
	PMT	=	45	45
	FV	=	1000	1000
Output:	I	=		

Similarly for YTC.