

Stock (Equity) Valuation

Issue

What is a share of stock worth?

Characteristics of Stock

Stockholders receive dividends and/or capital gains

Type	Dividend	Maturity	Priority
Preferred	Fixed (deferrable)	Perpetual or Fixed	Greater
Common	Variable (residual)	Perpetual	Lesser

Common stockholders are the true owners of the firm

Elect board of directors

Own all residual cash flows and value

Preferred stock is a hybrid of Debt and Common

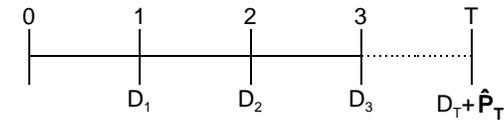
Claim is fixed and legally prior to common (like debt)

Dividend may be deferred without default (like common)

Maturity may be indefinite (like common)

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Expected Cash Flows to Common Shareholders



Value of Common Stock

PV of expected future cash flows

Dividends

Directly from firm

Paid out of earnings (rest is "retained")

Terminal Price

Received when sell shares

Paid by buyer

Determines capital gain

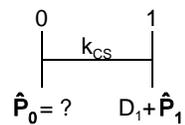
Depends on what *buyer thinks dividends will be* beyond that point

⇒ Expected capital gain reflects expected future dividends (beyond sale date)

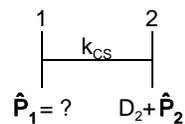
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Stock Values and Investors' Horizons

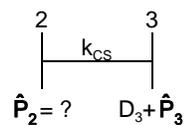
Investor's horizon has *no bearing* on stock's value



$$\hat{P}_0 = \frac{D_1}{1 + k_{CS}} + \frac{\hat{P}_1}{1 + k_{CS}}$$



$$\hat{P}_1 = \frac{D_2}{1 + k_{CS}} + \frac{\hat{P}_2}{1 + k_{CS}}$$



$$\hat{P}_2 = \frac{D_3}{1 + k_{CS}} + \frac{\hat{P}_3}{1 + k_{CS}}$$

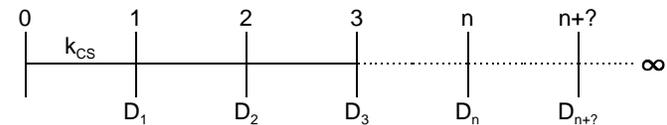
Whatever the planned holding period, investors care about *all* dividends, because even dividends beyond planned sale date help determine terminal price

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Dividends and Stock Value

Dividends: Basis of Value

Value is PV of all dividends that are *ever* expected



$$\hat{P}_0 = \frac{D_1}{1 + k_{CS}} + \frac{D_2}{(1 + k_{CS})^2} + \frac{D_3}{(1 + k_{CS})^3} + \dots$$

$$= \sum_{t=1}^{\infty} \frac{D_t}{(1 + k_{CS})^t}$$

where k_{CS} is the required rate of return for (this) common stock, from CAPM

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Constant Dividend Growth

Simplify

Don't need to know each dividend, if know their starting level and *relation between* them

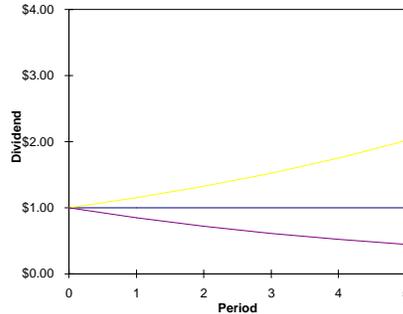
If the last dividend paid was D_0 and dividends are expected to grow from that level at an annual rate of g percent, then

$$D_1 = D_0 (1 + g)$$

$$D_2 = D_1 (1 + g) = D_0 (1 + g)^2$$

$$D_3 = D_2 (1 + g) = D_0 (1 + g)^3$$

...



Can estimate *any* dividend as

$$D_t = D_0 (1 + g)^t$$

Note: g an *average*

Constant Dividend Growth: Value

Non-Zero Growth (Gordon Model)

Applies best to large, mature firms

$$\hat{P}_0 = \frac{D_1}{k_{CS} - g} = \frac{D_0 (1 + g)}{k_{CS} - g}$$

as long as g is expected to last *forever* and $g \leq k_{CS}$, where k_{CS} is the required rate of return

Note: need only first future dividend in perpetually growing stream (D_1)

Zero Expected Growth

When $g = 0$, dividend stream a perpetuity

$$\hat{P}_0 = \frac{D}{k_{CS}}$$

Note: though stream infinite, value of distant dividends diminishes quickly

Digression: Sustainable Growth Rate

Definition

Maximum dividend growth rate firm can maintain indefinitely, without resorting to outside financing

$$g^* = b(r)$$

where

b = retention ratio (i.e., proportion of earnings not paid out as dividends or 1 - dividend payout ratio)

r = expected return on equity

A firm can grow its dividends faster
the more earnings it retains (the less it pays out as dividends)
the more it can earn on retained and reinvested earnings

Note: retention justified only if firm can earn more than investors

Constant Dividend Growth: Expected Return

Non-Zero Growth

$$\hat{k}_{CS} = \frac{D_1}{P_0} + g$$

= Dividend Yield + Capital Gains Yield

since expected Capital Gains Yield = $\frac{\hat{P}_1 - \hat{P}_0}{\hat{P}_0} = \frac{\frac{D_1(1+g)}{k_{CS}-g} - \frac{D_0(1+g)}{k_{CS}-g}}{\frac{D_0(1+g)}{k_{CS}-g}} = \frac{D_1 - D_0}{D_0} = g$

Zero Expected Growth

$$\hat{k}_{CS} = \frac{D}{P_0}$$

= Dividend Yield

Effective Return

Since dividends are usually paid quarterly, $m = 4$

Dividend, Capital Gain, and Total Returns

Dividend Yield (DY₁)

Annual dividend payment, expressed as a percentage of current price

$$DY_1 = \frac{D_1}{P_0}$$

Capital Gains Yield (CGY₁)

Annual change in stock price (at a given required return), expressed as a percentage of current price

$$CGY_1 = \frac{\hat{P}_1 - P_0}{P_0}$$

Total Return (TR₁)

Dividend plus capital gain, expressed as a percentage of current price
In equilibrium, equal to required return

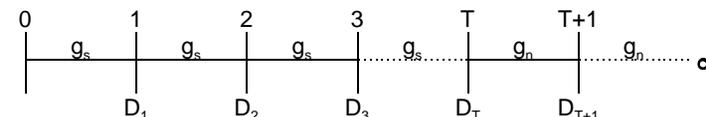
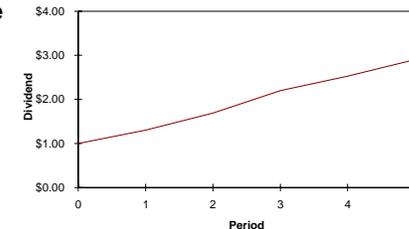
$$TR_1 = \frac{D_1 + \hat{P}_1 - P_0}{P_0} = DY_1 + CGY_1$$

Non-Constant Growth: Value

Dividend Growth Rate May Change over Time

Grow at one rate, g_s , for T years (g_s may $> k_{CS}$), then to change to sustainable long-run rate, g_n

Best fits start-ups, which can grow rapidly in early years, until competition catches up, when they become constant-growth firms



Value

$$\hat{P}_0 = \sum_{t=1}^T \frac{D_0 (1+g_s)^t}{(1+k_{CS})^t} + \frac{D_0 (1+g_s)^T (1+g_n)}{(k_{CS}-g_n) (1+k_{CS})^T}$$

Non-Constant Growth: Value (Details)

Handle piecewise: value early, non-constant-growth dividends and later constant-growth dividends separately, then sum

PV of Non-Constant Dividends (Part A)

$$PV(\text{Part A}) = \sum_{t=1}^T \frac{D_0 (1+g_s)^t}{(1+k_{CS})^t} = \frac{D_0 (1+g_s)}{1+k_{CS}} + \frac{D_0 (1+g_s)^2}{(1+k_{CS})^2} + \dots + \frac{D_0 (1+g_s)^T}{(1+k_{CS})^T}$$

PV of Constantly Growing Stream (Part B)

$$PV(\text{Part B}) = \frac{\hat{P}_T}{(1+k_{CS})^T} = \frac{\frac{D_{T+1}}{k_{CS}-g_n}}{(1+k_{CS})^T}$$

because, when apply constant-growth (Gordon) model to first dividend in constantly growing stream, $D_{T+1} = D_T (1+g_n)$, the result, $\hat{P}_T = \frac{D_{T+1}}{k_{CS}-g_n}$, is not a true PV

Non-Constant Growth: Expected Return

No Closed-form Solution

Like YTM, but more complicated
Requires trial-and-error

$$P_0 = \sum_{t=1}^T \frac{D_t}{(1+\hat{k}_{CS})^t} + \frac{D_{T+1}}{\hat{k}_{CS}-g_n} \left[\frac{1}{1+\hat{k}_{CS}} \right]^T$$

Dividend, Capital Gain, and Total Returns

Change over time, converging on stable, long-run values only when dividends reach constant-growth stage

However, in equilibrium, total return is equal to required return

Stock Market Equilibrium

Returns

Expected Return (from forecast) = Required Return (from CAPM)

$$\hat{k}_{CS} = \frac{D_1}{P_0} + g = k_{RF} + \beta_S (k_M - k_{RF}) = k_{CS}$$

Prices

Price (observed) = Value (from preferences)

$$P_0 = \hat{P}_0$$

Marginal Investor's Actions and their Results

if $\hat{k}_{CS} < k_{CS}$ and $P_0 > \hat{P}_0$, then sell, causing P_0 to fall

if $\hat{k}_{CS} > k_{CS}$ and $P_0 < \hat{P}_0$, then buy, causing P_0 to rise

Equilibrium

Marginal investor will hold only when $\hat{k}_{CS} = k_{CS}$ and $P_0 = \hat{P}_0$

Only then will prices be stable

Changes in prices are often large and rapid (see Market Efficiency)

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Preferred Stock: Value

Dividends

Do *not* grow

Value: Perpetual Preferred

like consol

$$\hat{P}_{0,PS} = \frac{D_{PS}}{k_{PS}}$$

Value: Sinking Fund Preferred

like coupon bond

$$\hat{P}_{0,PS} = \sum_{t=1}^n \frac{D_{PS}}{(1+k_{PS})^t} + \frac{M}{(1+k_{PS})^n}$$

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Preferred Stock: Expected Return

Perpetual Preferred

$$\hat{k}_{PS} = \frac{D_{PS}}{P_{0,PS}}$$

Sinking Fund Preferred

like YTM

$$P_{0,PS} = \sum_{t=1}^n \frac{D_{PS}}{(1+\hat{k}_{PS})^t} + \frac{M}{(1+\hat{k}_{PS})^n}$$

Effective Return

Since dividends are usually paid quarterly, $m = 4$

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