

Time Value of Money

- Issue
 - A dollar today is worth more than a dollar tomorrow (“Time Preference”)
 - Why?
 - How much more?
- Importance
 - Determines asset value, e.g., for common stock
 - Size
 - *Timing*
 - Risk

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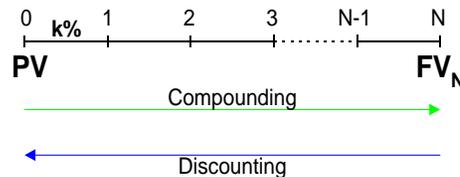
Cash Flow Patterns

- Single Cash Flow
 - lump sum (PV, FV)
- Annuity
 - level, regular stream (PMT)
- Perpetuity
 - perpetual annuity (PMT)
- Uneven Stream
 - CF_t

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Tools: Time Lines

- Cash Flows’
 - Size
 - Direction
 - Timing
- Knowns
- Unknowns



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Timelines: Practice

- Sketch timelines illustrating
 - \$100 to be received at the end of Year 2
 - a 3-year ordinary annuity of \$100
 - \$50 paid now, \$100 received in 1 year, \$75 received in 2 years, and \$50 received in 3 years

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Tools: Calculator

Variables in Problem

- Number of Periods
- Interest per Period
- Present Value (at $t = 0$)
- Size of Annuity (**PMT**)
- Future Value (at $t = N$)

N **I/YR** **PV** **PMT** **FV**

Note: Assign values to variables from timeline: first *number*, then **key**

Notes:
Set **P/YR** to 1
Set **BEG/END** to END

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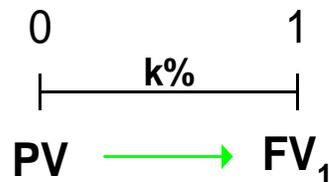
Single CFs: Characteristics

- Future Value (FV)
 - How much will something grow to be worth?
- Present Value (PV)
 - How much is some future CF worth now?
- Rate of Return or Yield (k , r or i)
 - How fast is something earning/growing?
- Number of Periods (N)
 - How long to reach target?

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Single CF: FV (1 period)

- Bank offers CD
 - Deposit
 - Maturity
 - Rate
- What's its payoff?

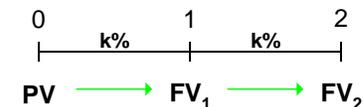


$$\begin{aligned} \text{Payoff} &= \text{Principal} + \text{Interest} \\ FV_1 &= PV + k(PV) \\ &= PV(1 + k) \end{aligned}$$

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Single CF: FV (2 periods)

- If roll over, what's payoff in 2 years?

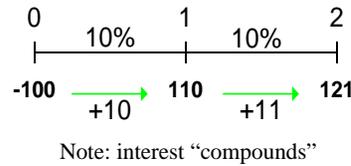


$$\begin{aligned} \text{Payoff} &= \text{Principal} + \text{Interest} \\ FV_2 &= FV_1(1 + k) \\ &= PV(1 + k)^2 \end{aligned}$$

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Single CF: FV (Example)

- Find FV of \$100 deposit earning 10% for
 - 1 year
 - 2 years

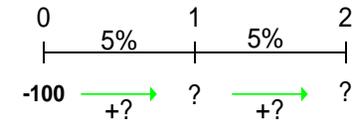


Inputs	1	10	-100		
	N	I/YR	PV	PMT	FV
Output					110.00
Inputs	2	10	-100		
	N	I/YR	PV	PMT	FV
Output					121.00

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Single CF: FV (Practice)

- Find FV of \$100 deposit earning 5% for
 - 1 year
 - 2 years



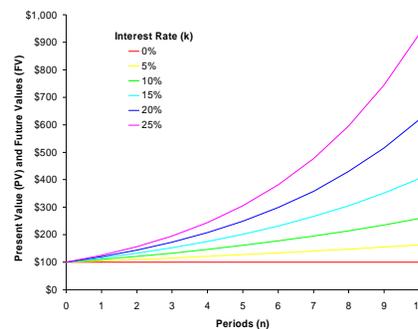
Inputs	1	5	-100		
	N	I/YR	PV	PMT	FV
Output					?
Inputs	2	5	-100		
	N	I/YR	PV	PMT	FV
Output					?

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Single CF: FV (Relationships)

- You are indifferent between PV and FV
- Given PV, FV is *larger*
 - the larger k
 - the larger n

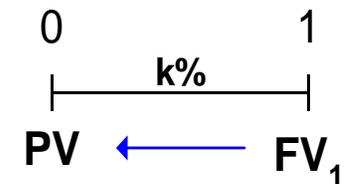
$$FV_n = PV(1 + k)^n$$



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Single CF: PV

- Investment promises FV_1 in 1 year
- If pass, you can earn k% (opportunity cost)
- What's it worth *now*?
 - What's the *most* you should pay for it?

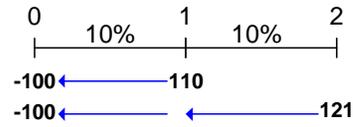


$$PV = \frac{FV_1}{1 + k}$$

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Single CF: PV (Example)

- If opportunity cost is 10%, find PV of
 - \$110 received in 1 year
 - \$121 received in 2 years

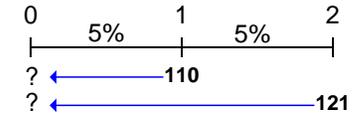


Inputs	1	10			110
	N	I/YR	PV	PMT	FV
Output			-100.00		

Inputs	2	10			121
	N	I/YR	PV	PMT	FV
Output			-100.00		

Single CF: PV (Practice)

- If opportunity cost is 5%, find PV of
 - \$110 received in 1 year
 - \$121 received in 2 years



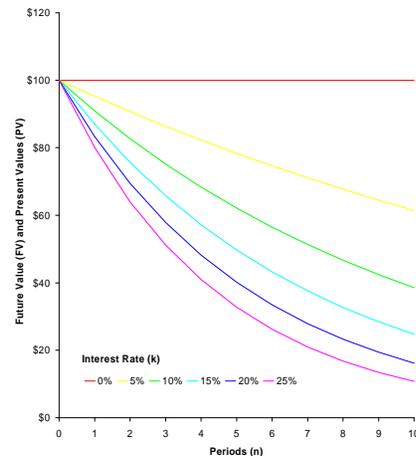
Inputs	1	5			110
	N	I/YR	PV	PMT	FV
Output			?		

Inputs	2	5			121
	N	I/YR	PV	PMT	FV
Output			?		

Single CF: PV (Relationships)

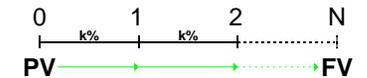
- You are indifferent between PV and FV
- Given FV, PV is *smaller*
 - the larger k
 - the larger n

$$PV = \frac{FV_n}{(1+k)^n}$$



Single CF: Rate

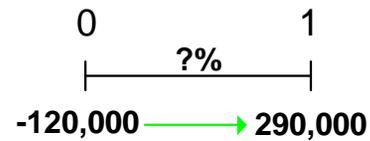
- What rate connects PV and FV over N periods?
 - Growth Rate
 - Rate of Return or Yield
- Often difficult to solve without financial calculator



$$FV_n = PV(1+k)^n$$

Growth Rate (Example)

- Last year, your firm sold \$120,000.
- This year you expect to have sold \$290,000.
- How fast did your sales grow during the year?



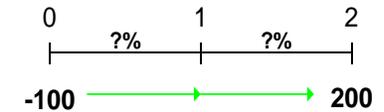
Inputs	1		-120000		290000
	N	I/YR	PV	PMT	FV
Output		141.67			

Note: sign change

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Rate of Return (Practice)

- You invested \$100 two years ago
- You just cashed out for \$200
- What rate of return (yield) did you earn?

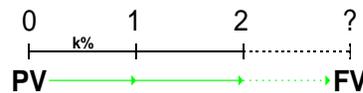


Inputs	2		-100		200
	N	I/YR	PV	PMT	FV
Output		?			

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Single CF: Periods

- How long will it take to grow PV into FV, if it grows at a periodic rate of k%?

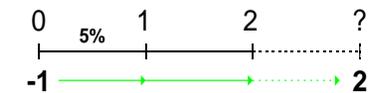


$$FV_n = PV(1 + k)^n$$

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Single CF: Periods (Practice)

- How long will it take to double your money, if you can earn 5% per year?



Inputs		5	-1		2
	N	I/YR	PV	PMT	FV
Output	?				

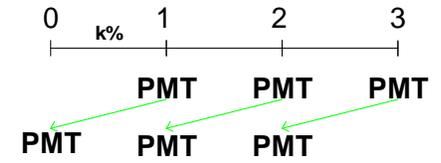
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Annuities: Characteristics

- Future Value (FV)
- Present Value (PV)
- Rate of Return or Yield (k, r or i)
- Number of Periods *and Payments* (N)
- Payment (PMT)

Types of Annuities

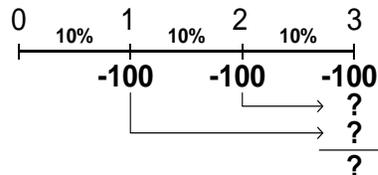
- Ordinary (deferred)
 - PMT at *end* of period
- Due (in advance)
 - PMT at *beginning* of period



Note: Set **BEG/END** key to BEG for Annuity Due, reset to END when finished

Ordinary Annuity: FV

- Deposit \$100 per year for 3 years, beginning in 1 year.
- If you can earn 10%, how much will you have in 3 years?

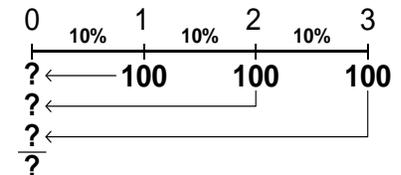


$$FVA = \sum_{t=1}^n PMT(1+k)^{n-t}$$

Inputs	3	10		-100	
	N	I/YR	PV	PMT	FV
Output					?

Ordinary Annuity: PV

- A contract will pay \$100 per year for 3 years, beginning in 1 year.
- If your opportunity cost is 10%, how much is it worth?



$$PVA = \sum_{t=1}^n \frac{PMT}{(1+k)^t}$$

Inputs	3	10		100	
	N	I/YR	PV	PMT	FV
Output			?		

Comparing CFs, PVs and FVs

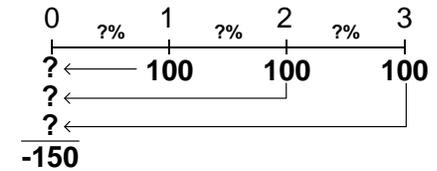
If your opportunity cost is 10%, would you rather have

- an annuity
 - \$100 per year for 3 years, starting in 1 year
- its PV
 - \$248.69 now
- or its FV?
 - \$331 in 3 years

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Ordinary Annuity: Rate

- A contract will pay \$100 per year for 3 years, beginning in 1 year.
- If it costs \$150, what is its rate of return?



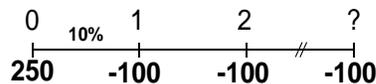
Inputs	3	-150	100	
	N	PV	PMT	FV
Output				?

Note: sign change

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Ordinary Annuity: Periods

- You borrowed \$250 at 10%.
- If you can repay \$100 per year, beginning in 1 year, how long to pay it off?



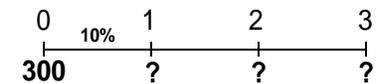
Inputs	10	250	-100	
	N	PV	PMT	FV
Output				?

Note: sign change

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Ordinary Annuity: Payment

- You borrowed \$300 at 10%.
- How much must you pay per year, beginning next year, in order to pay it off in exactly 3 years?

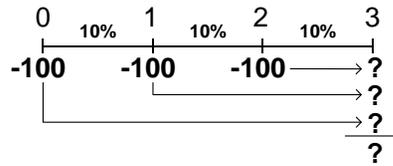


Inputs	3	10	300	
	N	I/YR	PV	FV
Output				?

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Annuity Due: FV

- Deposit \$100 per year for 3 years, beginning now.
- If you can earn 10%, how much will you have in 3 years?
- How compares to ordinary annuity?

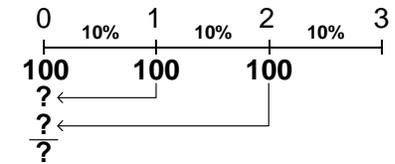


Inputs	3	10		-100	
	N	I/YR	PV	PMT	FV
Output					?

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Annuity Due: PV

- A contract pays \$100 per year for 3 years, beginning now.
- If your opportunity cost is 10%, how much is it worth?
- How compares to ordinary annuity?



Inputs	3	10		100	
	N	I/YR	PV	PMT	FV
Output			?		

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Comparing Annuities' Values

- FV
 - larger for annuity due than for otherwise identical ordinary annuity by 1 period's interest
- $$FVA_{due} = FVA_{ordinary}(1+k)$$
- PV
 - larger for annuity due than for otherwise identical ordinary annuity by 1 period's interest

$$PVA_{due} = PVA_{ordinary}(1+k)$$

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Annuity Due: Payment

- You want to accumulate \$400 within 3 years.
- You can earn 12% per year.
- How much must you invest each year, beginning now?



Inputs	3	12			400
	N	I/YR	PV	PMT	FV
Output				?	

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Perpetuities: Characteristics

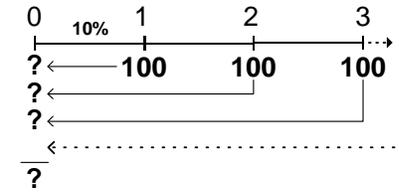
- Present Value (PV)
- Rate of Return or Yield (k, r or i)
- Payment (PMT)

Note: Functions *not* programmed into calculator

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Perpetuities: PV (Practice)

- A rich uncle has left you \$100 per year forever, beginning next year.
- You can earn 10% per year.
- For how much would you sell your inheritance?



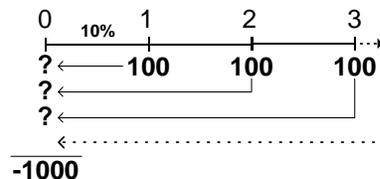
$$PV = \frac{PMT}{k}$$

Note: can approximate on calculator as annuity with *very* large N

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Perpetuities: Rate

- A contract will pay \$100 per year forever, beginning next year.
- If it costs \$1000, what is its rate of return?



$$k = \frac{PMT}{PV}$$

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Uneven Streams: Characteristics

- Future Value (FV)
- Present Value (PV)
- Rate of Return or Yield (k, r or i)
- Missing Value

Note: Use **CF**, **NPV**, and **IRR** keys

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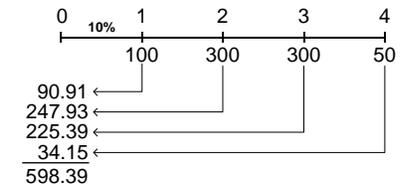
Uneven Streams: Special Keys

- **CF**
 - clear calculator/worksheet before starting
 - enter CFs, with sign, starting at Time 0
- **NPV**
 - finds PV
 - does *not* flip sign of result
- **IRR**
 - finds rate (otherwise difficult-to-impossible)

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Uneven Stream: PV (Example)

- You are offered this contract.
- Your opportunity cost is 10%.
- What's it worth?

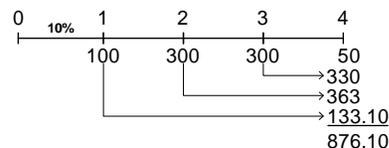


Note: $CF_0 = 0$

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Uneven Stream: FV (Example)

- You deposit money into an account.
- You can earn 10% per year.
- What will it grow to in 4 years?
 - Hint: Find **NPV**, then **FV**

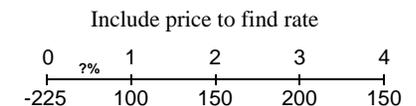
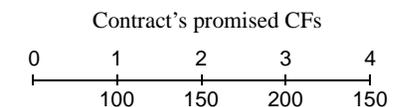


Note: $CF_0 = 0$

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Uneven Stream: Rate (Example)

- Someone offers to sell you this contract for \$225 today.
- What rate of return is offered?
 - Find **IRR**



Note: $CF_0 = -225$

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Non-Annual Compounding

- If interest compounded more than once per year, you
 - can earn more interest on interest (FV larger)
 - will forgo more interest on interest (PV smaller)

m periods per year
– 2 semiannually
– 4 quarterly
– 12 monthly
– 360/365 daily

Converting Inputs

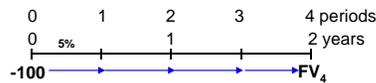
- Convert *recurring* items to periodic basis (e.g., monthly)
 - periodic rate
 - number of periods
 - payments (if any)

Conversion	for n years
Periods per Year	m
Periodic Rate	k/m
Number of Periods	mn
Periodic Payment	PMT/m



Semiannual: FV (Practice)

- You put \$100 into an account paying 10% per year, compounded semiannually.
- How much will you have in 2 years?
 - Hint: Like 4-year CD with 5% annual rate.

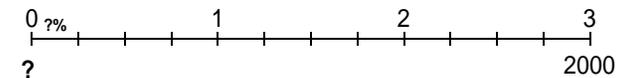


Convert	for 2 years
Periods per Year	2
Periodic Rate	5%
Number of Periods	4



Quarterly: PV (Practice)

- You are offered \$2000 in 3 years.
- If you can earn 8% per year, compounded quarterly, what is it worth today?



Non-Annual: FV (Relationships)

- Given PV, k, and n, FV is *larger*
 - the larger m
 - i.e., the more frequent compounding
 - or the shorter the interval between interest dates

$$FV_{mn} = PV \left(1 + \frac{k}{m} \right)^{mn}$$

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Non-Annual: PV (Relationships)

- Given FV, k, and n, PV is *smaller*
 - the larger m
 - i.e., the more frequent compounding
 - or the shorter the interval between interest dates

$$PV = \frac{FV_{mn}}{\left(1 + \frac{k}{m} \right)^{mn}}$$

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Effective Interest Rate (EAR)

- EAR adjusts all rates to *annual* basis (i.e., as if compounded once per year)
- EAR is rate which achieves same result (PV ↔ FV) as nominal rate plus compounding

$$EAR = \left(1 + \frac{k}{m} \right)^m - 1$$

Note: For $m > 1$, $EAR > k$

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EAR Calculation: Special Keys

HP calculators

- Special keys
 - P/YR**
 - number of compounding periods per year (m)
 - set to 1 for most purposes
 - NOM%**
 - Nominal or quoted rate (k or "APR")
 - EFF%**
 - EAR

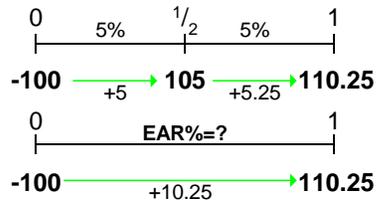
TI calculators

- ICONV Worksheet (2ND, 2)**
 - C/Y**
 - number of compounding periods per year (m)
 - NOM**
 - Nominal or quoted rate (k or "APR")
 - EFF**
 - EAR

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EAR (Example)

- If you can earn 10% per year, compounded semiannually, what *effective* rate can you earn?



$$EAR = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 0.1025$$

Inputs	10		2
	NOM%	EFF%	P/YR
Output		10.25	

Note: Reset **P/YR** to 1 when finished

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EAR (Practice)

- Find EAR, if you earn

– 10%, compounded annually

Inputs	10		1
	NOM%	EFF%	P/YR
Output		?	

– 12%, compounded quarterly

Inputs	12		4
	NOM%	EFF%	P/YR
Output		?	

– 5.25%, compounded daily

Inputs	5.25		365
	NOM%	EFF%	P/YR
Output		?	

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Types of Interest Rates

- Nominal or Quoted Rate (k , also APR)
 - stated in contracts
 - not meaningful without compounding periods
 - not used directly in calculations
- Periodic Rate (k/m)
 - actually charged/paid each period
 - generally used in calculation
 - less than nominal rate, unless $m = 1$
- Effective Rate (EAR)
 - same result as nominal rate, compounded
 - larger than nominal rate, unless $m = 1$

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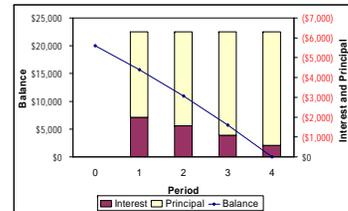
Amortized Loan

- Paid off with equal periodic payments, equally spaced (an annuity)
- Each payment includes both interest and principal, which change over time

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Amortized Loan (Example)

- You borrow \$20,000 at 10%, to be paid off over 4 years.
- How much interest and principal will you pay each year?



Period	Payment	Interest	Principal	Balance
0				\$20,000.00
1	(\$6,309.42)	(\$2,000.00)	(\$4,309.42)	\$15,690.58
2	(\$6,309.42)	(\$1,569.06)	(\$4,740.36)	\$10,950.22
3	(\$6,309.42)	(\$1,095.02)	(\$5,214.40)	\$5,735.82
4	(\$6,309.42)	(\$573.58)	(\$5,735.84)	(\$0.02)
	(\$25,237.68)	(\$5,237.66)	(\$20,000.02)	

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Amortized Loan: Calculator

- Input terms of loan (PV, k, n, m, FV)
- Find PMT
- Press **AMORT** key
- Set period(s)
- To see Interest, Principal, Balance results
 - HP: press = key repeatedly
 - TI: scroll down/up

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