# Capital Budgeting

**Decision**

Planned Expenditure on Long-term Assets (including intangibles)

**Importance**

Strategic—determines industry

Survival and Growth

Prolonged consequences

Large expenditures—outside financing

**Purpose**

Maximize shareholder wealth by investing in all projects which are *worth more than they cost*, i.e., for which

$$PV(benefits) > PV(costs)$$

**Process**

Parallel to security valuation:

1. Estimate cash flows
2. Estimate their risk and determine required return
3. Compare PVs of inflows and outflows

# Decision Rules

**Requirements**

To be worth using, a rule must:

- consider *all* cash flows and *only* cash flows
- use appropriate discount rate (from market)
- properly handle *mutually exclusive* projects

**Selected Rules**

Payback Period

Discounted Payback Period

Net Present Value (NPV)

Profitability Index (PI)

Internal Rate of Return (IRR)

Modified Internal Rate of Return (MIRR)

**Other Uses for these Techniques**

Mergers and Acquisitions

Spinoffs (Disinvestment)

Bond Refunding

Lease-Purchase Analysis

# Payback Period

$$0 = \sum_{t=0}^{PP} CF_t$$

| | Expected Cash Flows | | Cumulative Cash Flow | |
|------|-----------|-----------|-----------|-----------|
| Year | Project A | Project B | Project A | Project B |
| 0 | (10,000) | (10,000) | (10,000) | (10,000) |
| 1 | 6,000 | 5,000 | (4,000) | (5,000) |
| 2 | 4,000 | 5,000 | 0 | 0 |
| 3 | 3,000 | | 3,000 | 0 |
| 4 | 2,000 | | 5,000 | 0 |
| 5 | 1,000 | | 6,000 | 0 |

# Discounted Payback Period

$$0 = \sum_{t=0}^{DPP} \frac{CF_t}{(1 + k)^t}$$

| Year | Expected Cash Flows | | Cumulative PV | |
|------|-----------|-----------|-----------|-----------|
|      | Project A | Project B | Project A | Project B |
| 0 | (10,000) | (10,000) | | |
| 1 | 6,000 | 5,000 | | |
| 2 | 4,000 | 5,000 | | |
| 3 | 3,000 | | | |
| 4 | 2,000 | | | |
| 5 | 1,000 | | | |

# Net Present Value (NPV)

$$\textbf{NPV} = \text{PV}(\textit{inflows}) - \text{PV}(\textit{outflows})$$

$$= \sum_{t=0}^{n} \frac{CF_t}{(1+k)^t}$$

$$= -CF_0 + \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t}$$

**Decision Rule**

Accept project if its NPV $\geq 0$

# Internal Rate of Return (IRR)

IRR is that (hypothetical) discount rate which forces

PV(*inflows*) = PV(*outflows*)

$$0 = NPV$$

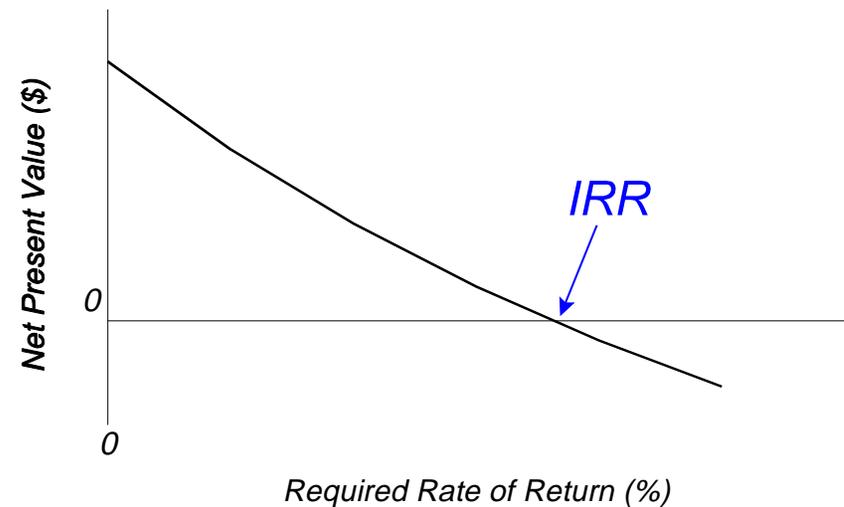$$0 = \sum_{t=0}^{n} \frac{CF_t}{(1+\textbf{IRR})^t}$$

$$CF_0 = \sum_{t=1}^{n} \frac{CF_t}{(1+\textbf{IRR})^t}$$

*IRR and NPV*



**Decision Rule**
Accept project if its IRR $\geq$ MCC

# Modified Internal Rate of Return (MIRR)

MIRR is that (hypothetical) discount rate which forces

$$PV\,(costs) = PV\,(terminal\ value)$$

$$\sum_{t=0}^{n} \frac{COF_t}{(1+k)^t} = \frac{\sum_{t=0}^{n} CIF_t(1+k)^{n-t}}{(1+\mathbf{MIRR})^n}$$

$$CF_0 = \frac{\sum_{t=1}^{n} CIF_t(1+k)^{n-t}}{(1+\mathbf{MIRR})^n}$$

**Decision Rule**
Accept project if its MIRR $\geq$ MCC

# Example Data

Required Rate of Return: 10%

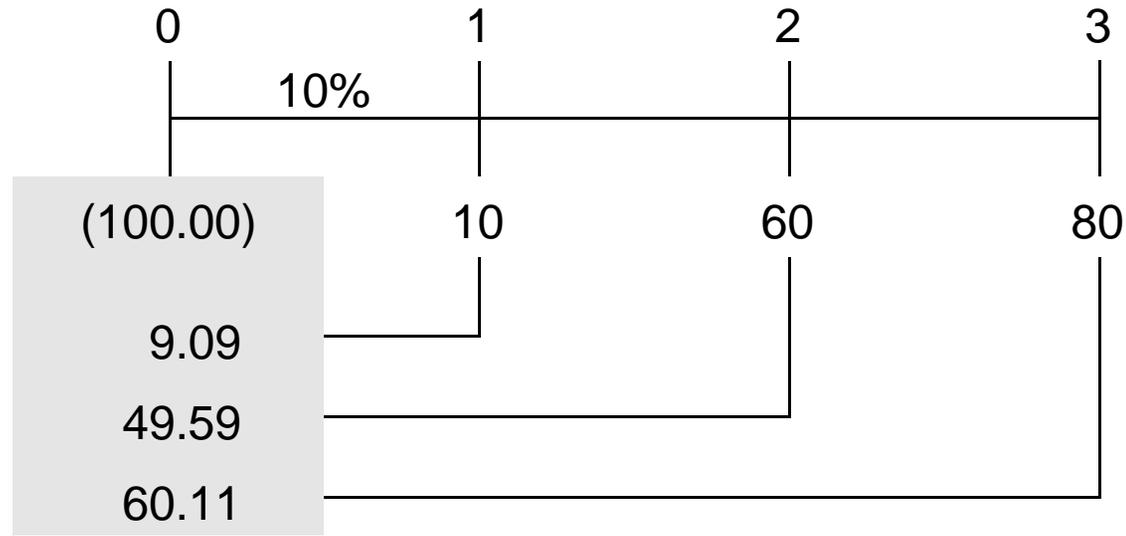| Year | Expected Cash Flows | |
| | Project L | Project S |
|---|---|---|
| 0 | (100) | (100) |
| 1 | 10 | 70 |
| 2 | 60 | 50 |
| 3 | 80 | 20 |

# Practice: NPV

**Project L**

|  | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 10% | | | | |
| | (100.00) | 10 | 60 | 80 |
| | 9.09 | | | |
| | 49.59 | | | |
| | 60.11 | | | |

**Decisions**

|  | Project L | Project S |
|---|---|---|
| NPV | | |
| Accept? | | |

# Practice: IRR

**Project L**

```
        0              1              2              3
        |──── 10% ─────|──────────────|──────────────|
    ┌─────────┐
    │(100.00) │        10             60             80
    │         │
    │   ?     │────────┘              │              │
    │         │                       │              │
    │   ?     │───────────────────────┘              │
    │         │                                      │
    │   ?     │──────────────────────────────────────┘
    └─────────┘
       0.00
```
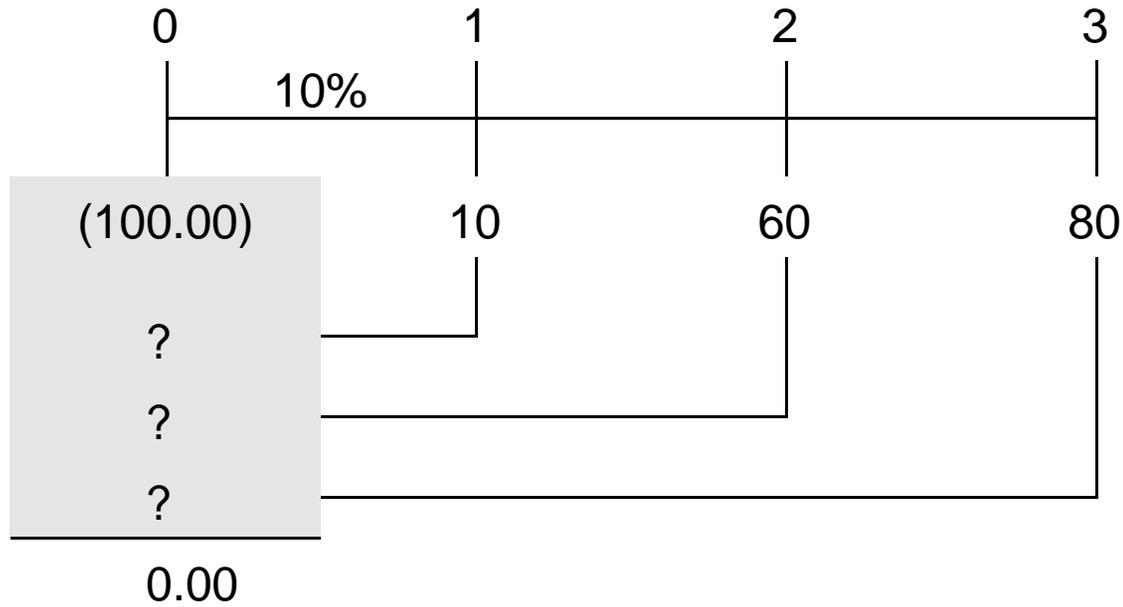
**Decisions**

|         | Project L | Project S |
|---------|-----------|-----------|
| IRR     |           |           |
| Accept? |           |           |

# Practice: MIRR

**Project L**

```
        0            1            2            3
        |───10%──────|────────────|────────────|
    (100.00)         10           60         80.00

                                             66.00

                                             12.10

                                            158.10

     100.00

      0.00
```
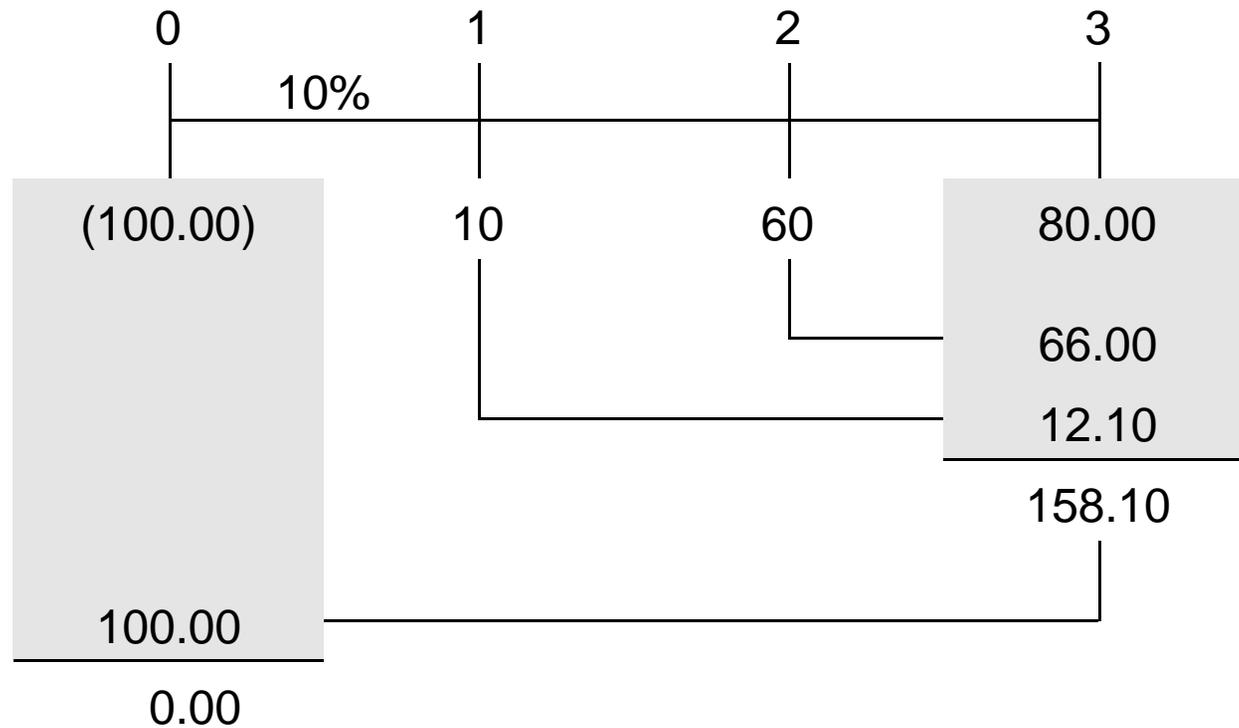
**Decisions**

|                | Project L | Project S |
|----------------|-----------|-----------|
| PV (Costs)     |           |           |
| TV$_3$         |           |           |
| MIRR           |           |           |
| Accept?        |           |           |

# NPV Profiles
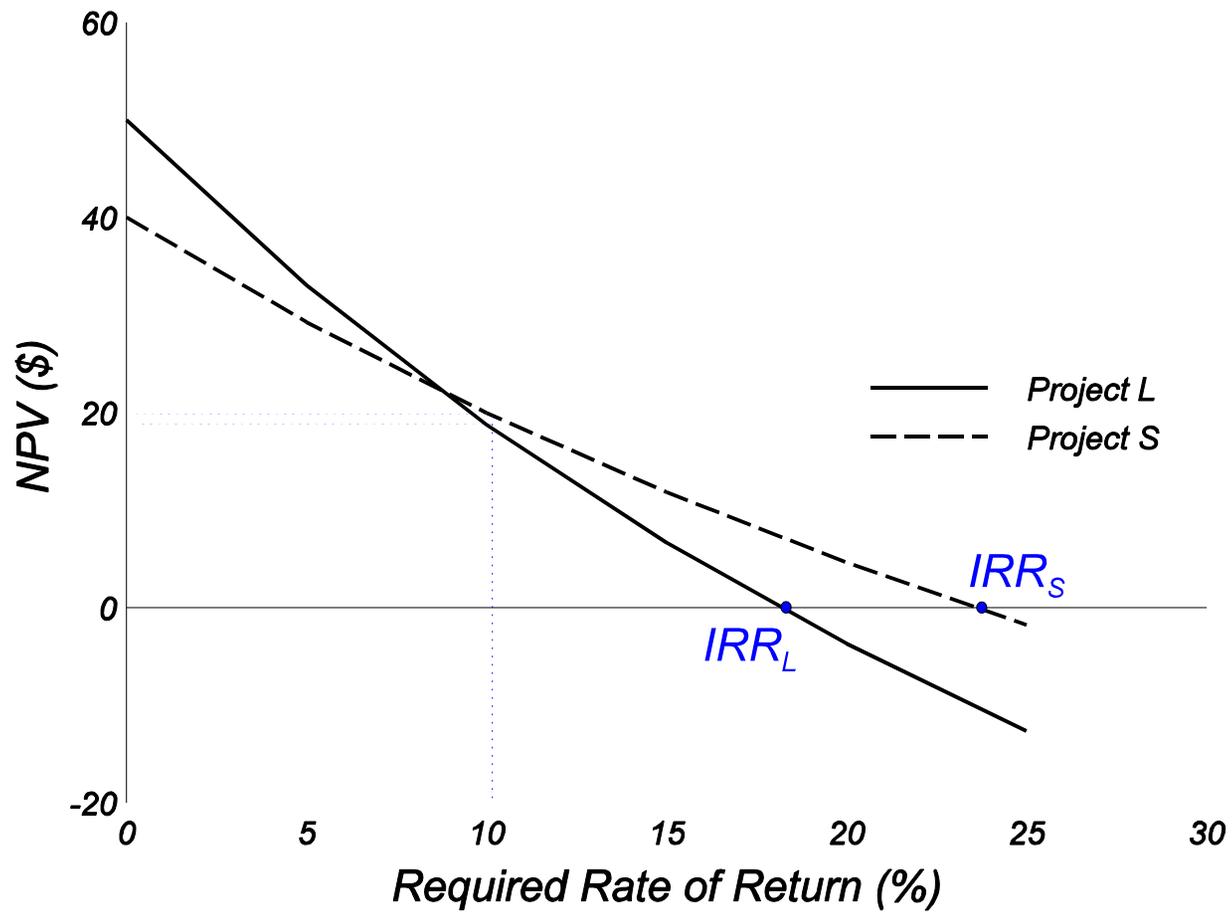
# Relations between Decision Methods

$$NPV \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \leftrightarrow IRR \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} k \leftrightarrow PI \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1$$

**Independent projects**
- Good/Bad $\Rightarrow$ Accept/Reject
- NPV, IRR, and PI give same decision

**Mutually exclusive projects**
- Good/Better $\Rightarrow$ Rank
- NPV, IRR, and PI *may* give conflicting decisions, if projects differ in:

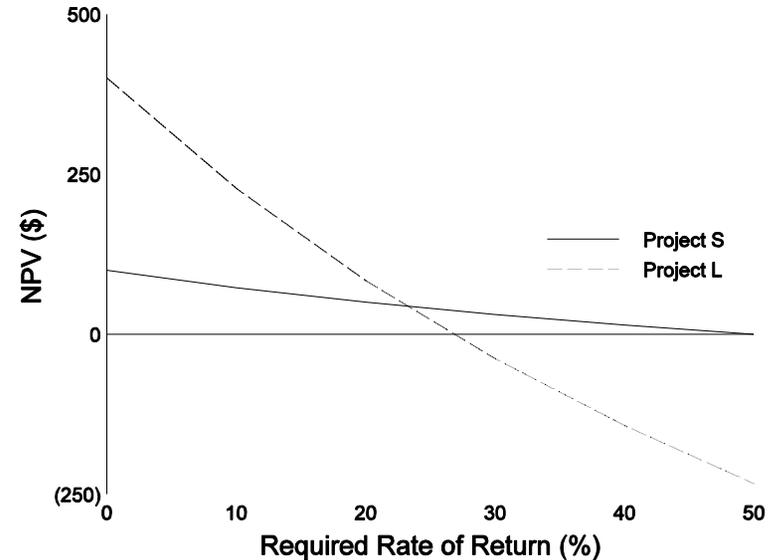| | |
|---|---|
| Scale | Size of investment |
| Timing | Early vs. late payoffs |
| Lives | Number of cash flows |

# Mutual Exclusion: Differences in Scale

k = 10%

| Year | Project S | Project L | Project Δ |
|------|-----------|-----------|-----------|
| 0 | (200) | (1,500) | (1,300) |
| 1 | 300 | 1,900 | 1,600 |
| NPV | $72.73 | $227.27 | $154.55 |
| PI | 1.36 | 1.15 | 1.12 |
| IRR | 50.00% | 26.67% | 23.08% |
| Accept? | | | |

**Possible Conflict**

Between:   NPV vs. PI, IRR

Cause:        absolute ($) vs. relative (%) measures of benefit

Remedies:  NPV, "Project Δ"
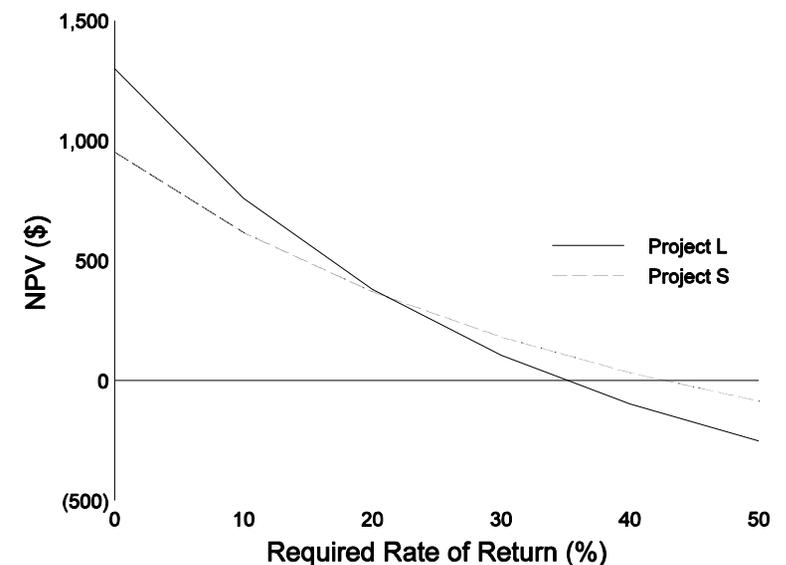
# Mutual Exclusion: Differences in Timing

k = 10%

| Year | Project L | Project S | Project Δ |
|------|-----------|-----------|-----------|
| 0 | (1,000) | (1,000) | 0 |
| 1 | 100 | 650 | (550) |
| 2 | 200 | 650 | (450) |
| 3 | 2,000 | 650 | 1,350 |
| NPV | $758.83 | $616.45 | $142.37 |
| PI | 1.76 | 1.62 | 1.16 |
| IRR | 34.84% | 42.57% | 21.01% |
| MIRR | 32.78% | 29.10% | 15.69% |
| Accept? | | | |

**Possible Conflict**

Between:    NPV, PI vs. IRR

Cause:    different reinvestment rates (MCC or IRR)

Remedies:  NPV, "Project Δ," MIRR

# Mutual Exclusion: Differences in Lives

| Year | Project S | Project L |
|------|-----------|-----------|
| k = 10% | | |
| 0 | (1,000) | (1,000) |
| 1 | 500 | 300 |
| 2 | 500 | 300 |
| 3 | 500 | 300 |
| 4 | | 300 |
| 5 | | 300 |
| 6 | | 300 |
| NPV | $243.43 | $306.58 |
| Accept? | | |

**Possible Conflict**

Between:    NPV vs. NPV

Cause:        Different reinvestment opportunities

Remedies:  Replacement Chain, Equivalent Annual Annuity
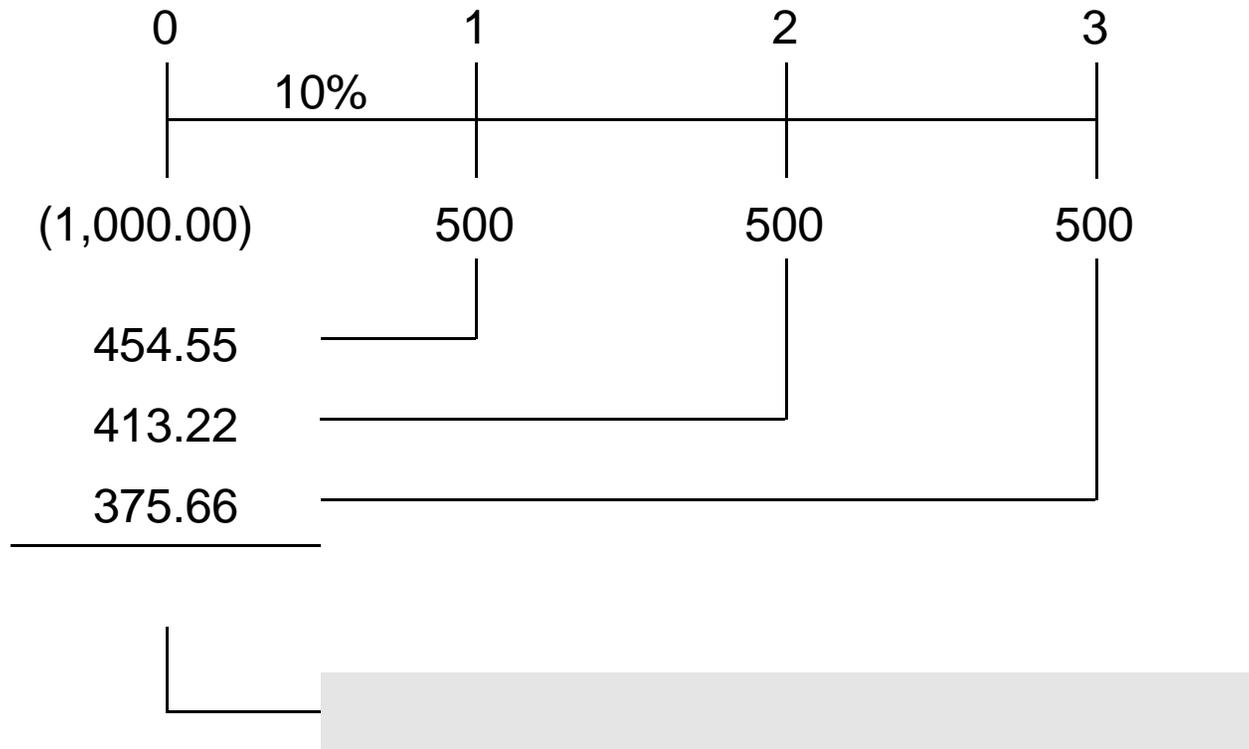
# Different Lives: Replacement Chain

k = 10%

| Year | Project S Original | Project S Replacement | Project S Combined | Project L |
|------|-----------|-------------|----------|-----------|
| 0 | (1,000) | | (1,000) | (1,000) |
| 1 | 500 | | 500 | 300 |
| 2 | 500 | | 500 | 300 |
| 3 | 500 | (1,000) | (500) | 300 |
| 4 | | 500 | 500 | 300 |
| 5 | | 500 | 500 | 300 |
| 6 | | 500 | 500 | 300 |
| NPV | $243.43 | $182.89 | $426.32 | $306.58 |
| Accept? | | | | |

# Different Lives: Equivalent Annual Annuity (EAA)

**Project S**

```
           0           1           2           3
           |           |           |           |
           |    10%     |           |           |
       (1,000.00)      500         500         500

   454.55 _____|
   413.22 _____|
   375.66 _____|
   _____|
           |
           |_____
```

# Practice: EAA

k = 10%

| Year | Project S | Project L |
|------|-----------|-----------|
| 0 | (1,000) | (1,000) |
| 1 | 500 | 300 |
| 2 | 500 | 300 |
| 3 | 500 | 300 |
| 4 | | 300 |
| 5 | | 300 |
| 6 | | 300 |
| NPV | $243.43 | $306.58 |
| Life | 3 | 6 |
| EAA | | |
| Accept? | | |

# Summary: Ranking Mutually Exclusive Projects

| Case | Possible Conflict | Cause | Remedy |
|------|-------------------|-------|--------|
| Scale | NPV v. IRR | absolute v. relative measures | NPV, Δ project |
| Timing | NPV v. IRR | different reinvestment rates | NPV, Δ project, MIRR |
| Lives | NPV v. NPV | different reinvestment opportunities | replacement chain, EAA |

# Other Problems with IRR: Multiple IRRs

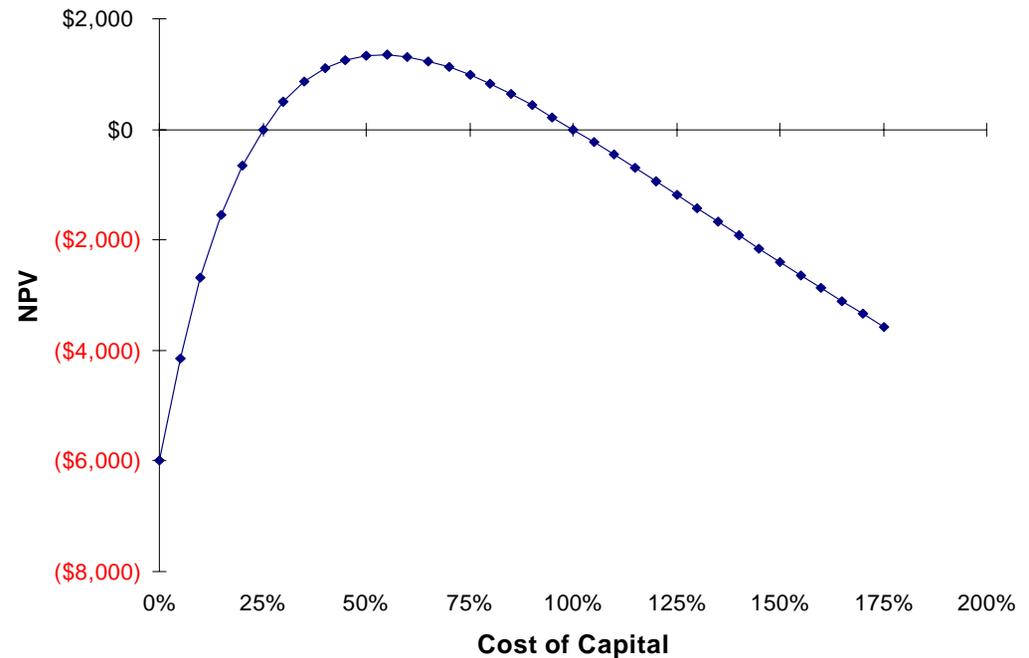**Problem and Remedy**

As many IRRs as sign changes in cash stream

IRR irrelevant

Use NPV or MIRR

**Example**

k = 10%

| Year | Cash Flow |
|------|-----------|
| 0 | (24,000) |
| 1 | 78,000 |
| 2 | (60,000) |
| NPV | ($2,677.69) |
| PI | 0.96 |
| IRR | |
| MIRR | 7.98% |

# Other Problems with IRR: Non-normal Projects

**Problem and Remedy**

IRR misleading

Must *reverse* IRR rule or use NPV

**Example**

k = 10%

| Year | Investment | Financing | Investment | Financing |
|------|-----------|-----------|-----------|-----------|
| 0 | (100) | 100 | (130) | 130 |
| 1 | 130 | (130) | 100 | (100) |
| NPV | $18.18 | ($18.18) | ($39.09) | $39.09 |
| PI | 1.18 | 0.85 | 0.70 | 1.43 |
| IRR | 30.00% | 30.00% | (23.08%) | (23.08%) |
| MIRR | 30.00% | (15.38%) | (23.08%) | 43.00% |