Capital Budgeting

Decision
Planned Expenditure on Long-term Assets (including intangibles)

Importance
Strategic—determines industry
Survival and Growth
Prolonged consequences
Large expenditures—outside financing

Purpose
Maximize shareholder wealth by investing in all projects which are *worth more than they cost*, i.e., for which

$$PV(benefits) > PV(costs)$$

Process
Parallel to security valuation:
1. Estimate cash flows
2. Estimate their risk and determine required return
3. Compare PVs of inflows and outflows

Decision Rules

Requirements
To be worth using, a rule must:
- consider *all* cash flows and *only* cash flows
- use appropriate discount rate (from market)
- properly handle *mutually exclusive* projects

Selected Rules
Payback Period
Discounted Payback Period
Net Present Value (NPV)
Profitability Index (PI)
Internal Rate of Return (IRR)
Modified Internal Rate of Return (MIRR)

Other Uses for these Techniques
Mergers and Acquisitions
Spinoffs (Disinvestment)
Bond Refunding
Lease-Purchase Analysis

Payback Period

$$0 = \sum_{t=0}^{PP} CF_t$$

	Expected Cash Flows		Cumulative Cash Flow	
Year	Project A	Project B	Project A	Project B
0	(10,000)	(10,000)	(10,000)	(10,000)
1	6,000	5,000	(4,000)	(5,000)
2	4,000	5,000	0	0
3	3,000		3,000	0
4	2,000		5,000	0
5	1,000		6,000	0

Discounted Payback Period

$$0 = \sum_{t=0}^{DPP} \frac{CF_t}{(1+k)^t}$$

	Expected Cash Flows		Cumulative PV	
Year	Project A	Project B	Project A	Project B
0	(10,000)	(10,000)		
1	6,000	5,000		
2	4,000	5,000		
3	3,000			
4	2,000			
5	1,000			

Net Present Value (NPV)

$$\textbf{NPV} = \text{PV}(\textit{inflows}) - \text{PV}(\textit{outflows})$$

$$= \sum_{t=0}^{n} \frac{CF_t}{(1+k)^t}$$

$$= -CF_0 + \sum_{t=1}^{n} \frac{CF_t}{(1+k)^t}$$

Decision Rule
Accept project if its NPV ≥ 0

Internal Rate of Return (IRR)
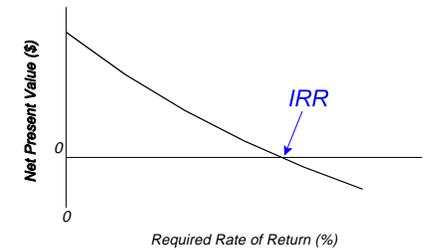
IRR is that (hypothetical) discount rate which forces

$$\text{PV}(\textit{inflows}) = \text{PV}(\textit{outflows})$$

$$0 = \text{NPV}$$

$$0 = \sum_{t=0}^{n} \frac{CF_t}{(1+\textbf{IRR})^t}$$

$$CF_0 = \sum_{t=1}^{n} \frac{CF_t}{(1+\textbf{IRR})^t}$$

IRR and NPV

Decision Rule
Accept project if its IRR ≥ MCC

Modified Internal Rate of Return (MIRR)

MIRR is that (hypothetical) discount rate which forces

$$\text{PV}(\textit{costs}) = \text{PV}(\textit{terminal value})$$

$$\sum_{t=0}^{n} \frac{COF_t}{(1+k)^t} = \frac{\sum_{t=0}^{n} CIF_t(1+k)^{n-t}}{(1+\textbf{MIRR})^n}$$

$$CF_0 = \frac{\sum_{t=1}^{n} CIF_t(1+k)^{n-t}}{(1+\textbf{MIRR})^n}$$

Decision Rule
Accept project if its MIRR ≥ MCC

Example Data

Required Rate of Return: 10%

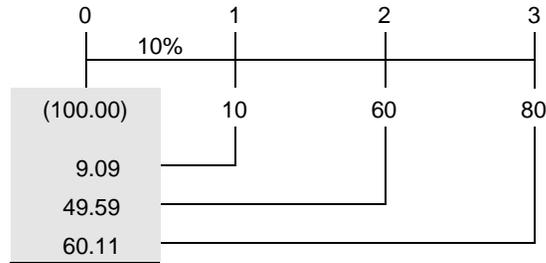|	Expected Cash Flows	
Year	Project L	Project S
0	(100)	(100)
1	10	70
2	60	50
3	80	20

Practice: NPV

Project L

```
        0           1           2           3
              10%
      (100.00)     10          60          80

        9.09
       49.59
       60.11
```

Decisions

	Project L	Project S
NPV		
Accept?		

Practice: IRR

Project L

```
        0           1           2           3
              10%
      (100.00)     10          60          80

         ?
         ?
         ?
        0.00
```
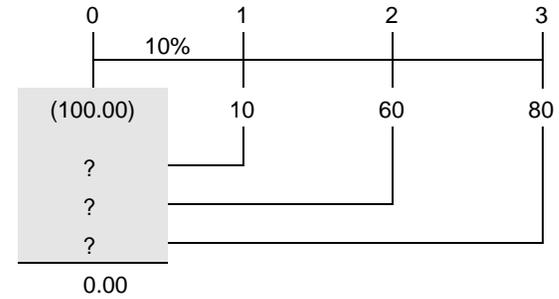
Decisions

	Project L	Project S
IRR		
Accept?		

Practice: MIRR

Project L

```
        0           1           2           3
              10%
      (100.00)     10          60         80.00

                                          66.00
                                          12.10
                                         158.10
       100.00
        0.00
```
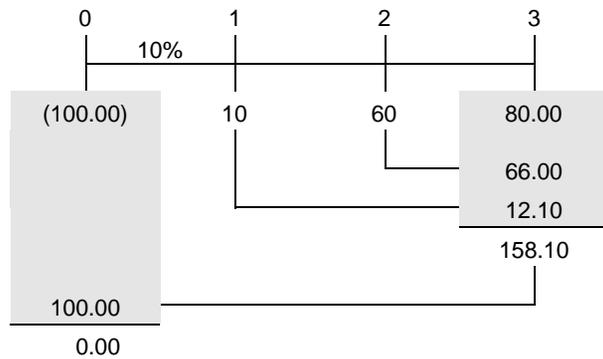
Decisions

	Project L	Project S
PV (Costs)		
TV_3		
MIRR		
Accept?		

NPV Profiles
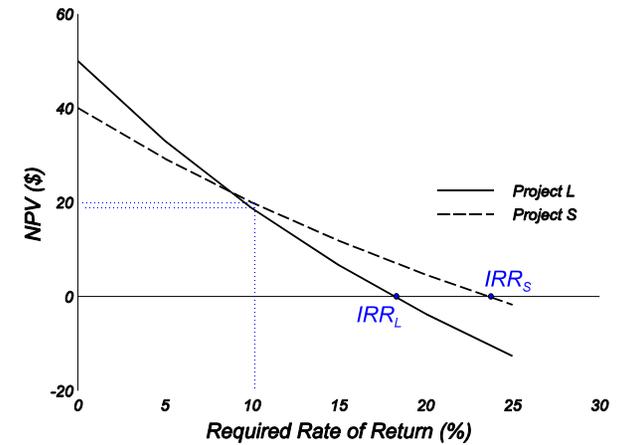
Relations between Decision Methods

$$\text{NPV} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \quad \text{IRR} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} k \quad \text{PI} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 1$$

Independent projects
- Good/Bad   Accept/Reject
- NPV, IRR, and PI give same decision

Mutually exclusive projects
- Good/Better   Rank
- NPV, IRR, and PI *may* give conflicting decisions, if projects differ in:

  Scale   Size of investment
  Timing   Early vs. late payoffs
  Lives   Number of cash flows
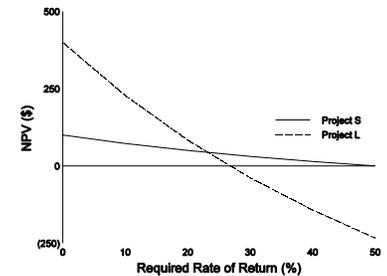
Mutual Exclusion: Differences in Scale

k = 10%

Year	Project S	Project L	Project Δ
0	(200)	(1,500)	(1,300)
1	300	1,900	1,600
NPV	$72.73	$227.27	$154.55
PI	1.36	1.15	1.12
IRR	50.00%	26.67%	23.08%
Accept?			

Possible Conflict

Between: NPV vs. PI, IRR
Cause: absolute ($) vs. relative (%) measures of benefit
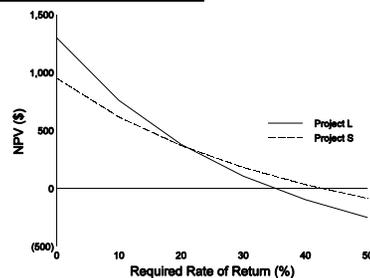Remedies: NPV, "Project Δ"

Mutual Exclusion: Differences in Timing

k = 10%

Year	Project L	Project S	Project Δ
0	(1,000)	(1,000)	0
1	100	650	(550)
2	200	650	(450)
3	2,000	650	1,350
NPV	$758.83	$616.45	$142.37
PI	1.76	1.62	1.16
IRR	34.84%	42.57%	21.01%
MIRR	32.78%	29.10%	15.69%
Accept?			

Possible Conflict

Between: NPV, PI vs. IRR
Cause: different reinvestment rates (MCC or IRR)
Remedies: NPV, "Project Δ," MIRR

Mutual Exclusion: Differences in Lives

k = 10%

Year	Project S	Project L
0	(1,000)	(1,000)
1	500	300
2	500	300
3	500	300
4		300
5		300
6		300
NPV	$243.43	$306.58
Accept?		

Possible Conflict

Between: NPV vs. NPV
Cause: Different reinvestment opportunities
Remedies: Replacement Chain, Equivalent Annual Annuity

Different Lives: Replacement Chain

k = 10%

	Project S			Project L
Year	Original	Replacement	Combined	
0	(1,000)		(1,000)	(1,000)
1	500		500	300
2	500		500	300
3	500	(1,000)	(500)	300
4		500	500	300
5		500	500	300
6		500	500	300
NPV	$243.43	$182.89	$426.32	$306.58
Accept?				

Different Lives: Equivalent Annual Annuity (EAA)

Project S

0	1	2	3
10%			
(1,000.00)	500	500	500

454.55
413.22
375.66
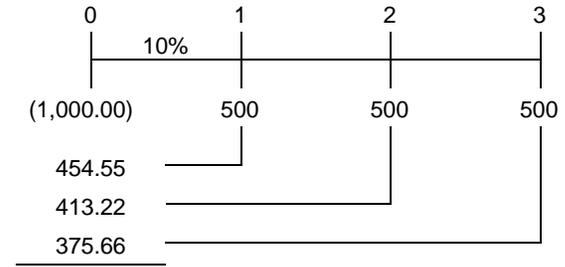
Practice: EAA

k = 10%

Year	Project S	Project L
0	(1,000)	(1,000)
1	500	300
2	500	300
3	500	300
4		300
5		300
6		300
NPV	$243.43	$306.58
Life	3	6
EAA		
Accept?		

Summary: Ranking Mutually Exclusive Projects

Case	Possible Conflict	Cause	Remedy
Scale	NPV v. IRR	absolute v. relative measures	NPV, Δ project
Timing	NPV v. IRR	different reinvestment rates	NPV, Δ project, MIRR
Lives	NPV v. NPV	different reinvestment opportunities	replacement chain, EAA

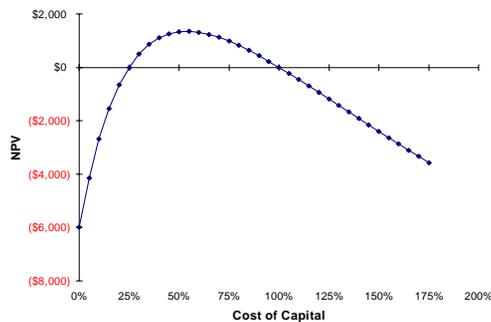## Other Problems with IRR: Multiple IRRs

Problem and Remedy
As many IRRs as sign changes in cash stream
IRR irrelevant
Use NPV or MIRR

Example

k = 10%

Year	Cash Flow
0	(24,000)
1	78,000
2	(60,000)
NPV	($2,677.69)
PI	0.96
IRR	
MIRR	7.98%

Other Problems with IRR: Non-normal Projects

Problem and Remedy
IRR misleading
Must *reverse* IRR rule or use NPV

Example

k = 10%

Year	Investment	Financing	Investment	Financing
0	(100)	100	(130)	130
1	130	(130)	100	(100)
NPV	$18.18	($18.18)	($39.09)	$39.09
PI	1.18	0.85	0.70	1.43
IRR	30.00%	30.00%	(23.08%)	(23.08%)
MIRR	30.00%	(15.38%)	(23.08%)	43.00%