Bonds

Definition
Long-term Debt: legally enforceable promise by borrower to pay specified amount(s) on specified date(s)

Contractual Characteristics
Maturity (N)
Par/Face/Maturity Value (FV)
Coupon Interest Rate, Coupon Payment (PMT)

$$\text{Coupon Payment} = \text{Coupon Interest Rate} \times \text{Par Value}$$

Security and Seniority of claim
Call Provisions: Call Premium/Price, Date of First Call/Protection Period, Sinking Fund
Conversion Terms
Indexing (e.g., TIPS)

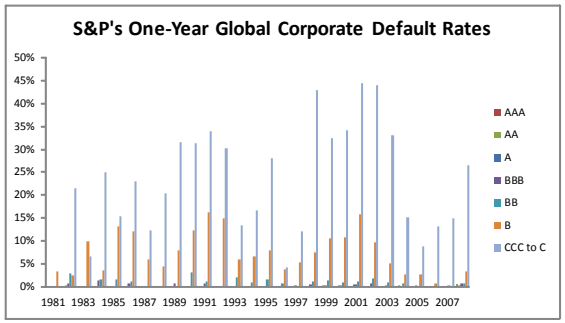
Risks
Default Risk
Interest Rate Risk (Price Risk, Risk to Wealth)
Reinvestment Rate Risk (Risk to Income)

Default Risk and Bond Ratings

Simplified Bond Ratings

	Investment Grade				Junk			
Moody's	Aaa	Aa	A	Baa	Ba	B	Caa	C
S&P	AAA	AA	A	BBB	BB	B	CCC	D

Actual Default Rates

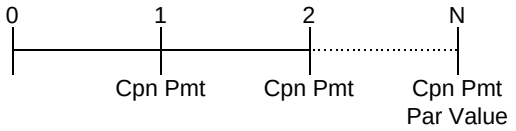


Digression: Liquidation ("Chapter 7")

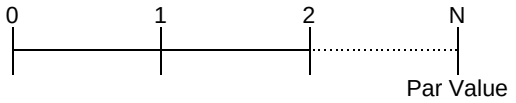
Priority of Payment
Debt owned by secured creditors (from sales of secured assets)
Trustee's costs
Wages (subject to limits)
Taxes
Unfunded pension liabilities
Debt owned by unsecured creditors (senior first, then junior)
Preferred stock
Common stock

Bonds' Cash Flow Patterns

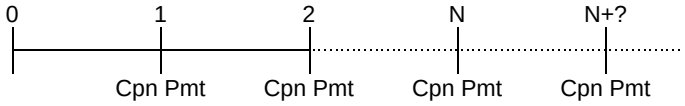
Coupon Bond



Zero-Coupon Bond

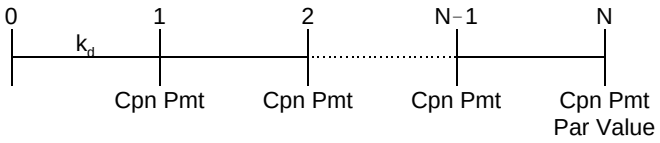


Perpetual Bond, e.g., "Consol"



Bond Valuation: Coupon Bond

Cash Flows



Value

$$\hat{P}_0 = \sum_{t=1}^{N} \frac{Cpn\ Pmt}{(1+k_d)^t} + \frac{Par\ Value}{(1+k_d)^N}$$

Inputs:	N	=	N
	I	=	k_d
	PMT	=	Cpn Pmt
	FV	=	Par Value
Output:	PV	=	$-\hat{P}_0$

where

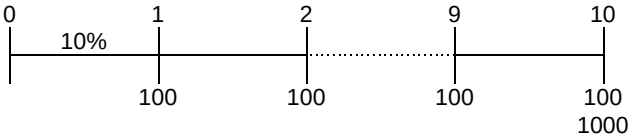
$$Cpn\ Pmt = Coupon\ Rate \times Par\ Value$$

$$k_{dl} = k_{RF} + DRP_l + LP_l + MRP_l$$

Coupon Bond Valuation: Practice 1

Example
Consider a 10-year bond with a $1,000 par value and a 10% annual coupon. If the interest rate is 10%, what's it worth?

Cash Flows



Value

$$P_0 = \sum_{t=1}^{10} \frac{100}{(1+0.10)^t} + \frac{1000}{(1+0.10)^{10}}$$

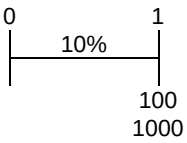
Calculator Solution

Inputs:	N	=	10
	I	=	10
	PMT	=	100
	FV	=	1000
Output:	PV	=	

Coupon Bond Valuation: Practice 2

Example
Consider a 1-year bond with a $1,000 par and a 10% annual coupon. If the interest rate is 10%, what's it worth?

Cash Flows



Value

$$P_0 = \sum_{t=1}^{1} \frac{100}{(1+0.10)^t} + \frac{1000}{(1+0.10)^1}$$

Calculator Solution

Inputs:	N	=	1
	I	=	10
	PMT	=	100
	FV	=	1000
Output:	PV	=	

Interest Rates, Maturity and Bond Values: Practice

Suppose the Interest Rate Increases to 13%

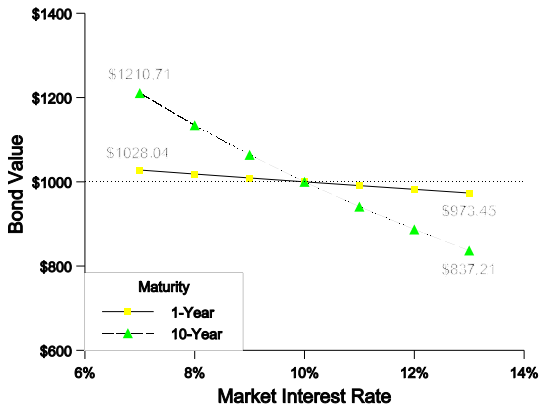
Bond			Long	Short
Inputs:	N	=	10	1
	I	=	13	13
	PMT	=	100	100
	FV	=	1000	1000
Output:	PV	=		

Suppose the Interest Rate Decreases to 7%

Bond			Long	Short
Inputs:	N	=	10	1
	I	=	7	7
	PMT	=	100	100
	FV	=	1000	1000
Output:	PV	=		

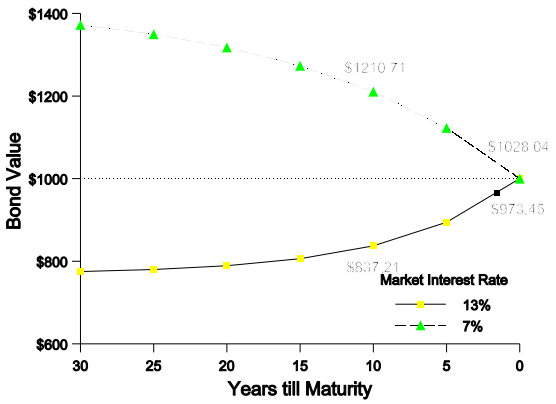
Interest Rates and Bond Value

Given Maturity



Maturity and Bond Value

Given the Interest Rate



Important Price-Yield Relationships

1. Bond values and interest rates are inversely related.

2. $$\text{Market Rate} \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} \text{Coupon Rate} \Leftrightarrow \text{Value} \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} \text{Par}$$

3. Interest rate risk is greater
   - the longer the maturity
   - the lower the coupon rate
   - the lower the initial yield

4. As a bond approaches maturity, its value approaches par.

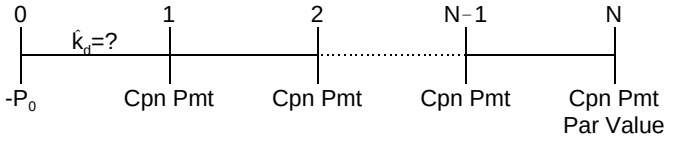
Yield to Maturity (YTM)

Definition
That discount rate which makes PV of promised CFs equal to bond's price, i.e.,

$$P_0 = \sum_{t=1}^{N} \frac{Cpn\ Pmt}{(1+\hat{k}_d)^t} + \frac{Par\ Value}{(1+\hat{k}_d)^N}$$

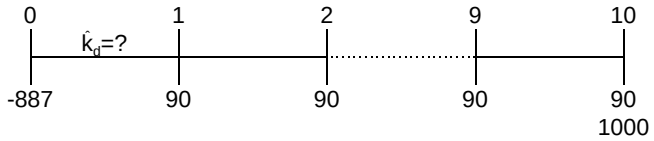
Interpretation
Promised return and, in equilibrium, *required* return or market rate

Illustration
If a bank offered the same cash flows (as the bond) for the same price (deposit), what rate of return is implied?



YTM: Practice

Discount Bond
A 9% coupon, 10-year, $1,000 par bond sells for $887. Find its YTM. Is it consistent with price-yield relationships?



Premium Bond
The same bond sells for $1,134.20. Find its YTM. Is it consistent with price-yield relationships?

Calculator Solution

Inputs:	N	=	10	10
	PV	=	-887	-1134.2
	PMT	=	90	90
	FV	=	1000	1000
Output:	I	=		

Current, Capital Gains, and Total Yields

Current Yield (CY)
Annual coupon interest payment, expressed as a percentage of current price

$$CY_1 = \frac{Cpn\ Pmt}{P_0}$$

Capital Gains Yield (CGY)
Annual change in bond price (at a given interest rate), expressed as a percentage of current price

$$CGY_1 = \frac{\hat{P}_1 - P_0}{P_0}$$

Total Yield (TY)
Interest plus capital gain, expressed as a percentage of current price
In equilibrium, equal to YTM

$$TY_1 = \frac{Cpn\ Pmt + \left(\hat{P}_1 - P_0\right)}{P_0} = CY_1 + CGY_1$$

Current, Capital Gains, and Total Yields: Practice

Discount Bond
A 9% coupon, 10-year, $1,000 par bond sells for $887. Find its Current Yield, Capital Gains Yield and Total Yield.

Premium Bond
The same bond sells for $1,134.20. Find its Current Yield, Capital Gains Yield and Total Yield.

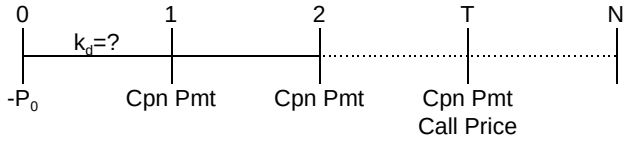
Calculator Solution

Cpn Pmt	=	90	90
P_0	=	887	1134.2
CY_1	=		
$\hat{P}_1$	=		
CGY_1	=		
TY_1	=		

Yield to First Call (YTC)

Concept
Since a bond may be called before its maturity, bondholder can't count on earning YTM. How much can he earn, if bond is called as soon as it becomes eligible for call?

Cash Flows

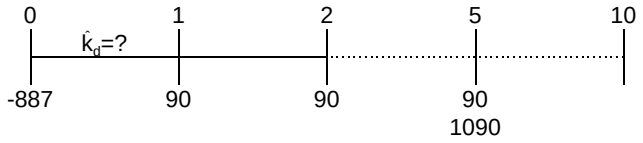


Calculation
Like YTM, after substituting date of first call (T) for maturity (N) and call price for par value

$$P_0 = \sum_{t=1}^{T} \frac{Cpn\ Pmt}{(1+\hat{k}_d)^t} + \frac{Call\ Price}{(1+\hat{k}_d)^T}$$

YTC: Practice

Discount Bond
A 9% coupon, 10-year, $1,000 par bond selling for $887, is callable in 5 years at a call price of $1,090. Find its YTC.



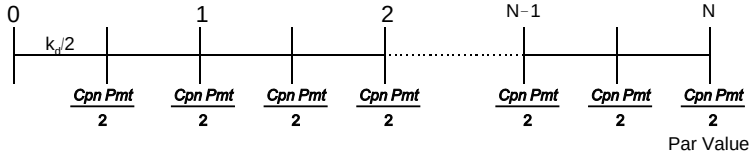
Premium Bond
The same bond sells for $1,134.20. Find its YTC.

Calculator Solution

Inputs:	N	=	5	5
	PV	=	-887	-1134.2
	PMT	=	90	90
	FV	=	1090	1090
Output:	I	=		

Semiannual Coupons

Cash Flows



Value

$$P_0 = \sum_{t=1}^{2N} \frac{\frac{\text{Cpn Pmt}}{2}}{\left(1+\frac{k_d}{2}\right)^t} + \frac{\text{Par Value}}{\left(1+\frac{k_d}{2}\right)^{2N}}$$

Calculator Solution

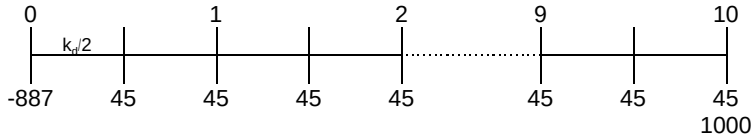
Inputs:	N	=	2N
	I	=	k_d/2
	PMT	=	Cpn Pmt/2
	FV	=	Par Value
Output:	PV	=	$-\hat{P}_0$

Semiannual Coupons: Value (Practice)

				Long	Short
Interest Rate = 10%	Bond				
	Inputs:	N	=	20	2
		I	=	5	5
		PMT	=	50	50
		FV	=	1000	1000
	Output:	PV	=		
Interest Rate = 13%	Bond				
	Inputs:	N	=	20	2
		I	=	6.5	6.5
		PMT	=	50	50
		FV	=	1000	1000
	Output:	PV	=		
Interest Rate = 7%	Bond				
	Inputs:	N	=	20	2
		I	=	3.5	3.5
		PMT	=	50	50
		FV	=	1000	1000
	Output:	PV	=		

Semiannual Coupons: YTM (Practice)

Discount Bond
A 9% semiannual-coupon, 10-year, $1,000 par bond sells for $887. Find its YTM. Is it consistent with price-yield relationships?



Premium Bond
The same bond sells for $1,134.20. Find its YTM. Is it consistent with price-yield relationships?

Calculator Solution

Inputs:	N	=	20	20
	PV	=	-887	-1134.2
	PMT	=	45	45
	FV	=	1000	1000
Output:	I	=		