

## Stock (Equity) Valuation

### Issue

What is a share of stock worth?

### Characteristics of Stock

Stockholders receive dividends and/or capital gains

Type	Dividend	Maturity	Priority
Preferred	Fixed (deferrable)	Perpetual or Fixed	Greater
Common	Variable (residual)	Perpetual	Lesser

Common stockholders are the true owners of the firm

Elect board of directors

Own all residual cash flows and value

Preferred stock is a hybrid of Debt and Common

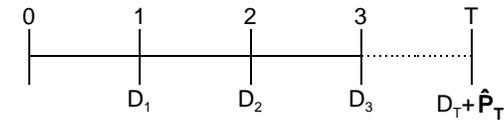
Claim is fixed and legally prior to common (like debt)

Dividend may be deferred without default (like common)

Maturity may be indefinite (like common)

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## Expected Cash Flows to Common Shareholders



### Value of Common Stock

PV of expected future cash flows

### Dividends

Directly from firm

Paid out of earnings (rest is "retained")

### Terminal Price

Received when sell shares

Paid by buyer

Determines capital gain

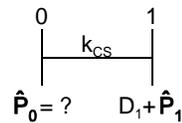
Depends on what *buyer thinks dividends will be* beyond that point

⇒ Expected capital gain reflects expected future dividends (beyond sale date)

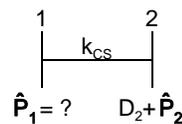
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## Stock Values and Investors' Horizons

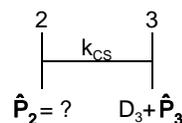
Investor's horizon has *no bearing* on stock's value



$$\hat{P}_0 = \frac{D_1}{1 + k_{CS}} + \frac{\hat{P}_1}{1 + k_{CS}}$$



$$\hat{P}_1 = \frac{D_2}{1 + k_{CS}} + \frac{\hat{P}_2}{1 + k_{CS}}$$



$$\hat{P}_2 = \frac{D_3}{1 + k_{CS}} + \frac{\hat{P}_3}{1 + k_{CS}}$$

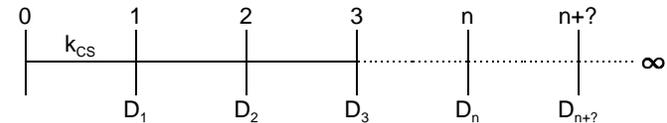
Whatever the planned holding period, investors care about *all* dividends, because even dividends beyond planned sale date help determine terminal price

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## Dividends and Stock Value

### Dividends: Basis of Value

Value is PV of all dividends that are *ever* expected



$$\hat{P}_0 = \frac{D_1}{1 + k_{CS}} + \frac{D_2}{(1 + k_{CS})^2} + \frac{D_3}{(1 + k_{CS})^3} + \dots$$

$$= \sum_{t=1}^{\infty} \frac{D_t}{(1 + k_{CS})^t}$$

where  $k_{CS}$  is the required rate of return for (this) common stock, from CAPM

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## Constant Dividend Growth

### Simplify

Don't need to know each dividend, if know their starting level and *relation between* them

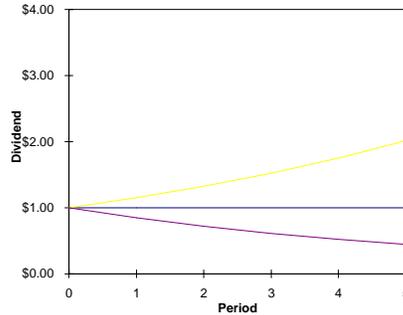
If the last dividend paid was  $D_0$  and dividends are expected to grow from that level at an annual rate of  $g$  percent, then

$$D_1 = D_0 (1 + g)$$

$$D_2 = D_1 (1 + g) = D_0 (1 + g)^2$$

$$D_3 = D_2 (1 + g) = D_0 (1 + g)^3$$

...



Can estimate *any* dividend as

$$D_t = D_0 (1 + g)^t$$

Note:  $g$  an *average*

## Constant Dividend Growth: Value

### Non-Zero Growth (Gordon Model)

Applies best to large, mature firms

$$\hat{P}_0 = \frac{D_1}{k_{CS} - g} = \frac{D_0 (1 + g)}{k_{CS} - g}$$

as long as  $g$  is expected to last *forever* and  $g \leq k_{CS}$ , where  $k_{CS}$  is the required rate of return

Note: need only first future dividend in perpetually growing stream ( $D_1$ )

### Zero Expected Growth

When  $g = 0$ , dividend stream a perpetuity

$$\hat{P}_0 = \frac{D}{k_{CS}}$$

Note: though stream infinite, value of distant dividends diminishes quickly

## Constant Dividend Growth: Expected Return

### Non-Zero Growth

$$\hat{k}_{CS} = \frac{D_1}{P_0} + g$$

= Dividend Yield + Capital Gains Yield

$$\text{since expected Capital Gains Yield} = \frac{\hat{P}_1 - \hat{P}_0}{\hat{P}_0} = \frac{\frac{D_1(1+g)}{k_{CS}-g} - \frac{D_0(1+g)}{k_{CS}-g}}{\frac{D_0(1+g)}{k_{CS}-g}} = \frac{D_1 - D_0}{D_0} = g$$

### Zero Expected Growth

$$\hat{k}_{CS} = \frac{D}{P_0}$$

= Dividend Yield

### Effective Return

Since dividends are usually paid quarterly,  $m = 4$

## Dividend, Capital Gain, and Total Returns

### Dividend Yield (DY<sub>t</sub>)

Annual dividend payment, expressed as a percentage of current price

$$DY_1 = \frac{D_1}{P_0}$$

### Capital Gains Yield (CGY<sub>t</sub>)

Annual change in stock price (at a given required return), expressed as a percentage of current price

$$CGY_1 = \frac{\hat{P}_1 - P_0}{P_0}$$

### Total Return (TR<sub>t</sub>)

Dividend plus capital gain, expressed as a percentage of current price  
In equilibrium, equal to required return

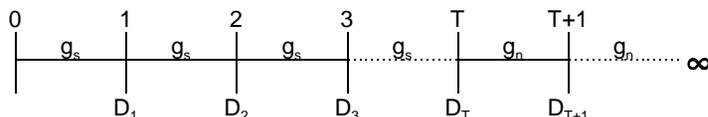
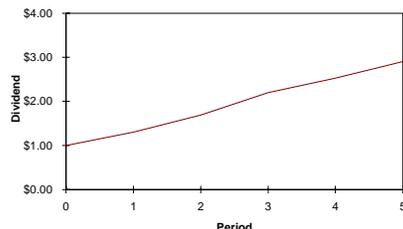
$$TR_1 = \frac{D_1 + \hat{P}_1 - P_0}{P_0} = DY_1 + CGY_1$$

## Non-Constant Growth: Value

### Dividend Growth Rate May Change over Time

Grow at one rate,  $g_s$ , for  $T$  years ( $g_s$  may  $> k_{CS}$ ), then to change to sustainable long-run rate,  $g_n$

Best fits start-ups, which can grow rapidly in early years, until competition catches up, when they become constant-growth firms



### Value

$$\hat{P}_0 = \sum_{t=1}^T \frac{D_0 (1+g_s)^t}{(1+k_{CS})^t} + \frac{D_0 (1+g_s)^T (1+g_n)}{(k_{CS}-g_n)(1+k_{CS})^T}$$

## Non-Constant Growth: Value (Details)

Handle piecewise: value early, non-constant-growth dividends and later constant-growth dividends separately, then sum

### PV of Non-Constant Dividends (Part A)

$$PV(\text{Part A}) = \sum_{t=1}^T \frac{D_0 (1+g_s)^t}{(1+k_{CS})^t} = \frac{D_0 (1+g_s)}{1+k_{CS}} + \frac{D_0 (1+g_s)^2}{(1+k_{CS})^2} + \dots + \frac{D_0 (1+g_s)^T}{(1+k_{CS})^T}$$

### PV of Constantly Growing Stream (Part B)

$$PV(\text{Part B}) = \frac{\hat{P}_T}{(1+k_{CS})^T} = \frac{\frac{D_{T+1}}{k_{CS}-g_n}}{(1+k_{CS})^T}$$

because, when apply constant-growth (Gordon) model to first dividend in constantly growing stream,  $D_{T+1} = D_T (1+g_n)$ , the result,  $\hat{P}_T = \frac{D_{T+1}}{k_{CS}-g_n}$ , is not a true PV

## Stock Market Equilibrium

### Returns

Expected Return (from forecast) = Required Return (from CAPM)

$$\hat{k}_{CS} = \frac{D_1}{P_0} + g = k_{RF} + \beta_S (k_M - k_{RF}) = k_{CS}$$

### Prices

Price (observed) = Value (from preferences)

$$P_0 = \hat{P}_0$$

### Marginal Investor's Actions and their Results

if  $\hat{k}_{CS} < k_{CS}$  and  $P_0 > \hat{P}_0$ , then sell, causing  $P_0$  to fall

if  $\hat{k}_{CS} > k_{CS}$  and  $P_0 < \hat{P}_0$ , then buy, causing  $P_0$  to rise

### Equilibrium

Marginal investor will hold only when  $\hat{k}_{CS} = k_{CS}$  and  $P_0 = \hat{P}_0$

Only then will prices be stable

Changes in prices are often large and rapid (see Market Efficiency)

## Preferred Stock: Value

### Dividends

Do *not* grow

### Value: Perpetual Preferred

like consol

$$\hat{P}_{0,PS} = \frac{D_{PS}}{k_{PS}}$$

### Value: Sinking Fund Preferred

like coupon bond

$$\hat{P}_{0,PS} = \sum_{t=1}^n \frac{D_{PS}}{(1+k_{PS})^t} + \frac{M}{(1+k_{PS})^n}$$

## Preferred Stock: Expected Return

### Perpetual Preferred

$$\hat{k}_{PS} = \frac{D_{PS}}{P_{0,PS}}$$

### Sinking Fund Preferred

like YTM

$$P_{0,PS} = \sum_{t=1}^n \frac{D_{PS}}{(1 + \hat{k}_{PS})^t} + \frac{M}{(1 + \hat{k}_{PS})^n}$$

### Effective Return

Since dividends are usually paid quarterly,  $m = 4$