

## L23: Sound Waves

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We have discussed the concept of forces and motion in this course, as well as concepts of energy and momentum. We have discussed special types of motion – in particular *simple harmonic motion* in which an object moves in a repetitive fashion characterized by a sinusoidal function.

We shall now conclude this course with a discussion of another special type of motion. The strange thing about the motion we are about to discuss is that what is probably the most obvious feature of this motion – a *traveling wave* – is not really a physical motion at all in the way we are used to speaking of it. That is, it does not involve an object or material moving from one place to another in the direction of the *apparent* wave motion. Instead, what we perceive as *wave motion* is really the result of the propagation of energy from one position to another within a *medium* (the material through which the energy signal is propagating). There are two media (plural of *medium*) in particular that we will be considering: waves traveling through *strings* and through *air*. The latter waves, which are really *pressure waves*, are what we more commonly call *sound waves*, and will be the focus of the end of this lecture.

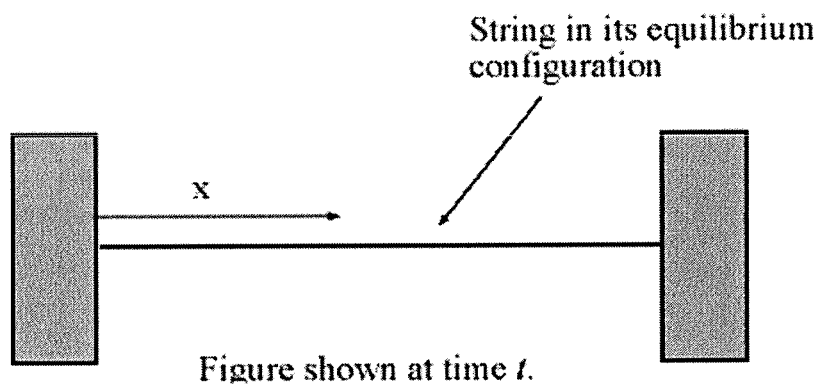
## Traveling Waves

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We can think of a *wave* as a traveling disturbance of a substance or material from its equilibrium configuration. This disturbance is *energy* being transferred from one part of the system to another.

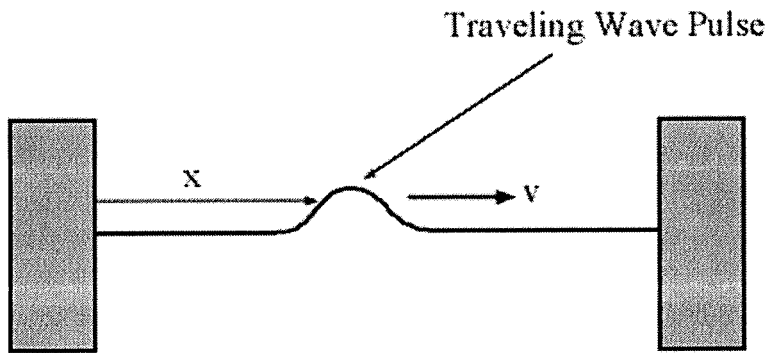
For example, the figure below shows a string (the *substance, material* or *medium* referred to above) stretched between two posts in its equilibrium configuration. We thus say that the displacement of any part of the string from its equilibrium configuration is *zero*. If we let the distance from the left end of the string be denoted  $x$ , then the displacement of any part of the system,  $y$ , at the position  $x$  and the time  $t$  is given (for this case of equilibrium) by

$$y(x,t) = 0 . \quad (\text{System in equilibrium configuration.})$$



All the equation above says is that, at any position  $x$  and at any time  $t$ , the displacement of the string from its equilibrium position is *zero* – that is, it is *in* its equilibrium position (just stretched horizontally with no wave pulse in it).

However, consider the case in which we pluck a part of the string. We have then caused a disturbance in the string from its equilibrium configuration. It took energy for us to displace the part of the string that we pluck from its equilibrium position – we transferred this energy to the string in the form of *work*. This energy is then transferred down the length of the string with a speed  $v$  which is characteristic of the physical properties of the medium through which the wave pulse is propagating (in this case, the string). More will be said about the wave speed in a later section of this lecture. The following figure shows this wave pulse moving toward the right end of the string.

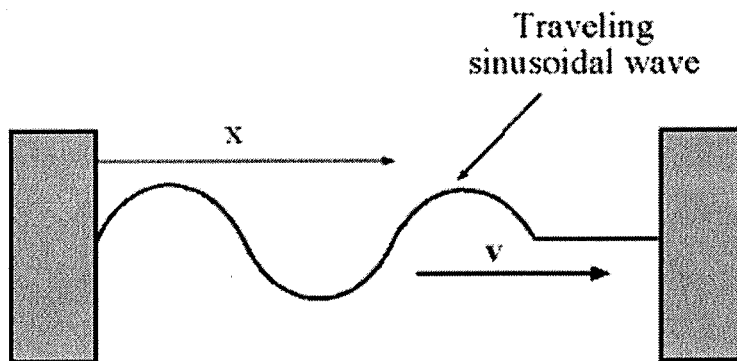
Figure shown at time  $t$ .

We can now see that, at a position  $x$  from the end of the string and at a time  $t$ , the displacement of the string from its equilibrium position is *not* necessarily zero:

$$y(x,t) \neq 0.$$

The exact form of the function  $y$  describes the *shape* of the wave pulse that is traveling down the string. The fact that the function  $y$  can depend on both the position  $x$  and the time  $t$  is necessary to describe a *traveling* wave, since, at any position  $x$ , the displacement  $y$  can vary with time as the wave travels by and, at any time  $t$ , the displacement from equilibrium can vary with the position  $x$ . The only possibility in describing a traveling wave is thus for  $y$  to depend on both  $x$  and  $t$ !

We shall consider only a very special and important case for the function  $y(x,t)$ : the special case called *harmonic traveling waves* or simply *sinusoidal waves*. These special waves form a continuous wave that has the form of a *sine-wave* – that is, a simple up-and-down pattern, as shown in the figure below.

Figure shown at time  $t$ .

The function describing this special type of traveling wave is given by

$$y(x,t) = y_{\max} \sin(kx - \omega t).$$

As with simple harmonic motion, the quantity  $y_{\max}$  represents the *amplitude* of the wave (that is, the maximum displacement from equilibrium). The quantity  $k$  is called the *wave number*, and  $\omega$  is the *angular frequency*. The fact that the *phase*, or argument of the sine function (that is, the quantity  $kx - \omega t$ ) has a *minus* sign in the middle of it tells us that the wave is traveling in the *positive- $x$*  direction. (If the phase had a *plus* sign in the middle instead of a *minus* sign,  $kx + \omega t$ , then the equation would be describing a wave traveling in the *negative- $x$*  direction – just the opposite of the sign in the middle!) We discuss the definitions of these and related quantities in the next section.

## Some Definitions

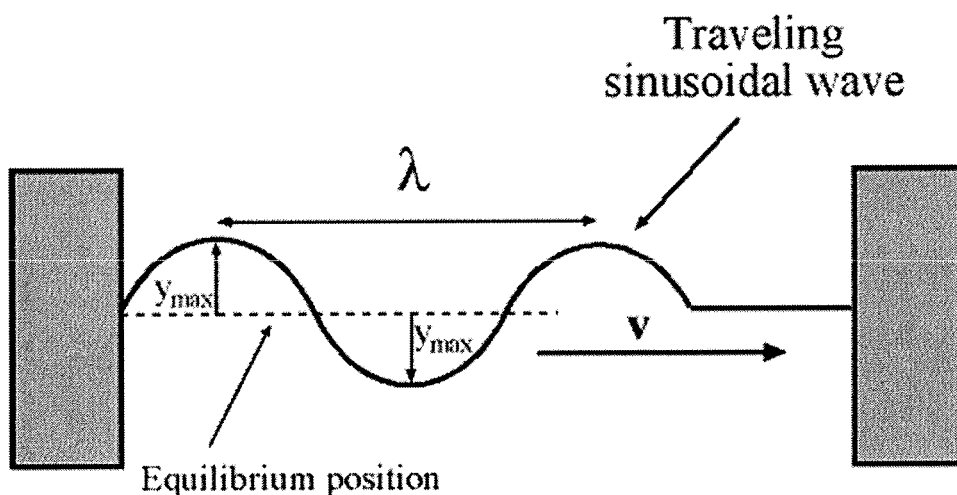
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We saw in the last section that a traveling sinusoidal wave has one of the following forms:

$$y(x,t) = y_{\max} \sin(kx - \omega t) \quad (\text{positive-}x \text{ motion})$$

$$y(x,t) = y_{\max} \sin(kx + \omega t) \quad (\text{negative-}x \text{ motion}).$$

We shall now discuss some of the definitions associated with these wave forms.



The figure above shows the sinusoidal traveling wave as it is just starting out. In principle, this sinusoidal traveling wave as described by the equation above would extend throughout the entire region of space where the wave is defined by the given equation (for example, perhaps just the region of space between the two supports in the figure above). The quantities listed in the previous section, along with some associated quantities, are listed, along with their MKS units, and defined in the table below.

Symbol	Name	Units	Comments/Relations
$y_{\max}$	amplitude	m	Gives the maximum magnitude of displacement from the equilibrium position
$\lambda$	wavelength	m	Gives the distance from one point on the wave to the next equivalent point
$k$	wavenumber	rad/m	$k = 2\pi/\lambda$

x	position	m	Gives the position along the medium through which the wave is propagating
T	period	s	Gives the amount of time for one wavelength to pass by a given point (that is, the time for one repetition or cycle of the wave)
f	linear frequency	Hz	Gives the number of wavelengths (or repetitions, cycles) passing by a given point per second: $f = 1/T$
$\omega$	angular frequency	rad/s	Gives the corresponding number of radians per second (1 cycle = $2\pi$ rad): $\omega = 2\pi f = 2\pi/T$
t	time	s	Time at which the wave is being observed at position $x$
v	wave speed	m/s	The speed with which the wave appears to be moving in the positive or negative $x$ direction: $v = \lambda/T = \omega/k$

You should commit to memory the definitions given above if you do not already know them (most of them are repeats from our previous studies of simple-harmonic motion and circular motion).

## Two Wave Types

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There are two types of traveling waves: *transverse waves* and *longitudinal waves*. These two wave types are characterized by the direction of the displacement of the medium through which the wave is propagating as the wave travels by a given point.

A *transverse wave* is the type of wave we've been discussing so far: the wave traveling through a string is an example of a transverse wave. Imagine drawing a black dot at a certain point on the string. As the wave travels by, you would see the dot move up and down (in the  $y$  direction; a *vertical* displacement from equilibrium, as shown in the figure in the previous section), *perpendicular* to the direction of motion of the wave (the  $x$  direction). Thus, the particle (or dot) displacement is *transverse* to the direction of wave motion.

On the other hand, a *longitudinal wave* is a wave in which the particles in the medium move *parallel* to the direction of motion of the wave (the  $x$  direction). A *sound wave* (in air or water) is an example of a longitudinal wave. As you speak, your lungs push air out of your mouth. This air pushes on the air in front of it, which pushes on the air in front of it – thereby causing a pressure wave whose oscillations (the moving back-and-forth of the air molecules) are along the same direction as the wave motion (from you to the person you are speaking to, for example).

No matter which type of wave we are speaking of, however, it is important to keep in mind that the form of the wave is still given by

$$y(x,t) = y_{\max} \sin(kx - \omega t) .$$

The only difference will be the meaning of the symbol  $y$  (it will mean the displacement *perpendicular* to the wave motion for a transverse wave, and the displacement from equilibrium *parallel* to the wave motion for a longitudinal wave).

## Wave Speeds

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As was mentioned previously, the speed of waves is determined by the physical properties of the medium through which the wave is moving – for example, how heavy the string is and how tightly it is stretched. We shall discuss and give relations for the speed of waves traveling in two types of media: waves in a string or rope, and waves traveling through air. You will have to be given the values of the speed of traveling waves in any other medium.

### Waves on Strings

Let's say that we have a string or rope of length  $L$  and total mass  $m$ . The **linear mass density** (or *mass-per-unit length*), denoted  $\mu$  (the Greek letter *mu*), is simply defined to be  $\mu = m/L$ . Let's further say that the string has been stretched somehow so that its tension is  $F_T$  (the tension is often imposed by a mass tied to the end of the string). It then follows that the speed of waves traveling on the string is given by

$$v = \sqrt{\frac{F_T}{\mu}}.$$

If the tension is in  $N$  and the linear mass density is in  $kg/m$ , then the wave speed will be in  $m/s$ . (Can you show this?)

### Waves in Air

The case of waves traveling through air (that is, **sound waves**) is a bit more complicated. We shall give two simple expressions below. First, the speed of sound in air depends on the *pressure* of the air through which the sound is moving. We will denote this pressure  $p$ . Second, it depends on the density of the air,  $\rho$  (the Greek letter *rho*). Lastly, the wave speed also depends on something from the thermodynamics of gasses called the **ratio of specific heats** (more specifically, the ratio of the specific heat of the air at constant pressure to that at constant volume,  $c_p/c_v$  – see a text on thermodynamics for more information), denoted  $\gamma$  (the Greek letter *gamma*). The equation for the wave speed in air (or any gas, for that matter) is

$$v = \sqrt{\frac{\gamma p}{\rho}}.$$

For air, we usually have that  $p = p_{\text{atm}} = 1.01 \times 10^5 \text{ Pa}$ ,  $\rho = 1.3 \text{ kg/m}^3$ , and  $\gamma = 7/5 = 1.4$  (no units). We would then get that  $v = 330 \text{ m/s}$ . Officially, at standard atmospheric conditions and  $0 \text{ }^\circ\text{C}$ , the speed of sound in air is equal to

$$v_{\text{air}} = 331 \text{ m/s} .$$

Of course, the pressure and density of the air can vary with many factors, in particular **temperature**. The approximate **temperature dependence** of the speed of sound in air is given by

$$v = \left( 331 \frac{\text{m}}{\text{s}} \right) \sqrt{1 + \frac{T}{273}} ,$$

where the temperature  $T$  is to be given in  $^\circ\text{C}$  (so that, at  $0 \text{ }^\circ\text{C}$ , the speed is  $331 \text{ m/s}$ , as mentioned above).

NOTE: You will **not** have to memorize the equations or numbers given in this section – they will be provided for you on quizzes or exams if you need them.

## Solution to Example 23.1

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A string has a length of **10 m** and a mass of **200 g**. A sinusoidal wave is imposed on the string after it is stretched such that the displacement of the string from its equilibrium position,  $y$ , as a function of the distance from one end,  $x$ , and the time,  $t$ , is given by  $y(x,t) = 0.020 \sin(4\pi x + 10\pi t)$ , where all numbers have MKS units.

Before we proceed with the solution, let's first write down all that we know so far. We are given that

$$L = 10 \text{ m} \quad m = 0.20 \text{ kg}$$

Furthermore, knowing that the generic equation for oscillatory traveling wave motion is  $y(x,t) = y_{\max} \sin(kx - \omega t)$ , we can see simply by comparison that (watch the units!)

$$y_{\max} = 0.020 \text{ m} \quad k = 4\pi \text{ rad/m} = 12.6 \text{ rad/m} \quad \omega = 10\pi \text{ rad/s} = 31.4 \text{ rad/s}$$

What is (a) the amplitude of the wave motion?

Simply by inspection of the given equation, we see that  $y_{\max} = 0.020 \text{ m}$ .

(b) the displacement of a string particle that is **0.73 m** from the  $x = 0$  end of the string at the time  $t = 1.3 \text{ s}$ ?

This is simply asking us to evaluate the equation given in the problem statement at  $x = 0.73 \text{ m}$  and  $t = 1.3 \text{ s}$ . While this may sound trivial (and it *is*!), there is an important point to this part of the problem. Namely, when evaluating the sine function in the equation for a traveling wave, you must be certain that your calculator is in **radians mode**. See your group members or instructor if you do not know how to put your calculator in radians mode, and then make sure that you know how to switch it back to degrees mode! We get that

$$\begin{aligned} y(x = 0.73 \text{ m}, t = 1.3 \text{ s}) &= 0.020 \sin(4\pi[0.73] + 10\pi[1.3]) \\ &= 0.020 \sin(50.0 \text{ rad}) \\ &= (0.020 \text{ m})(-0.262) \\ &= -0.00525 \text{ m} \\ &\cong -5.3 \text{ mm}. \end{aligned}$$

(c) the wavelength of the wave?

Since  $k = 12.6 \text{ rad/m}$  from above, and since  $k = 2\pi/\lambda$ , we get that  $\lambda = 2\pi/k = 0.50 \text{ m}$ .

(d) the time it takes for one wavelength to pass a given point?

This is just asking us for the *period* of the motion. Since  $\omega = 2\pi/T = 31.4 \text{ rad/s}$ , it follows that  $T = 2\pi/\omega = 0.20 \text{ s}$ .

(e) the wave speed?

At this point, we could either say that  $v = \lambda/T$ , or that  $v = \omega/k$  (you should be able to show from the definitions that these two expressions are equivalent to one another!). Either way we do it, we get that  $v = 2.5 \text{ m/s}$ .

(f) the direction of propagation of the wave?

The *plus sign* in the middle of the phase of the sine wave (the expression  $kx + \omega t$ ) tells us that the wave must be traveling in the *negative-x* direction (remember – it's always *opposite* to the sign in the middle of the phase!).

(g) the number of waves that pass by a given point each second?

The number of waves passing a given point per second is the definition of the frequency of the wave. We then immediately get that the linear frequency of the wave is  $f = 1/T = 5.0 \text{ Hz}$ . This says that there are **5.0** wavelengths (or cycles) that pass each point per second.

(h) the tension in the string?

There is only one expression that we have involving the string tension – that is the equation for the speed of the wave in a string:

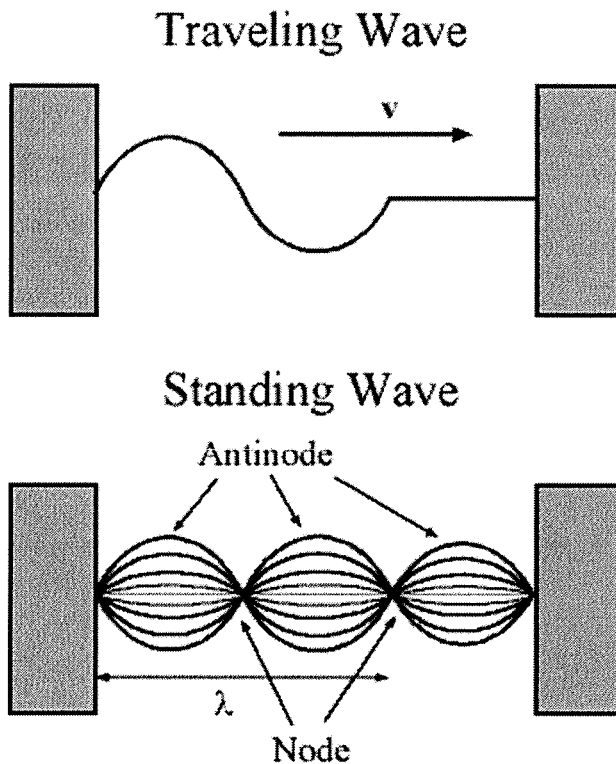
$$v = \sqrt{\frac{F_T}{\mu}}.$$

The linear mass density for this string is  $\mu = m/L = 0.020 \text{ kg/m}$ . It then follows that

$$F_T = \mu v^2 = 0.13 \text{ N}.$$

## Standing Waves

In general, when a wave travels through a given medium and “bumps” into another medium (such as a heavier string, or a wall), then part of the wave will be **transmitted** (for example, into the heavier rope) and part will be **reflected**. It is the **reflected** part that we are mainly interested in here.



Consider the figure at left. A traveling wave with a certain frequency has been produced at the left-hand end of the stretched string. The wave travels down to the right-hand end of the string, where it reflects off of the wall. If the traveling wave is still produced at the left end of the string, then the reflected wave traveling to the left from the right end of the string will combine or **superpose** with the original wave traveling toward the right from the left end of the string. If the conditions are just right, then the result of the superposition of the two waves is the **standing wave** shown in

the lower figure.

When there is a **standing wave** in a string all you see is one or more **bumps** (the figure above shows *three* bumps) which seem to oscillate up and down, but which don't seem to be moving toward the right or left (hence they seem to be **standing still**). Note that one **wavelength** of the wave that produced the standing waves corresponds to *two bumps*. The tops of the bumps are called **antinodes**, while the ends of the bumps (the points of zero motion) are called **nodes**.

When standing waves are produced, we say that a condition of **resonance** has been reached. The frequencies at which this resonance phenomenon occurs are called the **resonant frequencies** for that string or rope (the same applies, for example, to standing pressure waves of air in organ pipes or flutes). The smallest resonant frequency (corresponding to the largest wavelength) is called the **fundamental frequency**. Any other resonant frequency is always an integer multiple of the fundamental frequency. This integer is called the

*harmonic number*, and is denoted  $n$ .

In the standing wave shown in the figure above, for example, there are *three* antinodes and *four* nodes (including the two end points). The length of the string is thus equal to *three-halves* of a wavelength (since each *bump* corresponds to *one-half* of a wavelength). The number of bumps tells us the harmonic number – thus, for this case,  $n = 3$ . (One bump corresponds to the fundamental frequency. This is *three* times that frequency, or *one-third* the wavelength of the string vibrating in this fundamental mode. Remember that  $v = \lambda f$ . Thus, since  $v$  remains a constant for a fixed configuration of the string, if  $l$  is three times *smaller*, then  $f$  must be three times *larger*, so that  $n$  must be 3. – This is just another way of looking at it...!)

## Standing Waves in Strings

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In order to have a standing wave in a string, each of the two ends must be a *node* (as shown in the figure in the previous section). The number of *bumps* (or *antinodes*) tells us the *harmonic number*  $n$  for the standing wave. The harmonic number then tells us the frequency of the standing wave, which is given by the equation

$$f_n = \frac{nv}{2L},$$

where  $n$  is the harmonic number ( $n = 1, 2, 3, \dots$ ),  $v$  is the wave speed on the string, and  $L$  is the length of the string. Remember that, for a wave on a string, the wave speed is determined by the tension in the string and the linear mass density (mass per length) as given by

$$v = \sqrt{\frac{F_T}{\mu}}.$$

There is a certain terminology that is used in conjunction with these standing waves, especially when in association with music. (These standing waves on strings produce the musical notes heard from stringed instruments such as guitars, violins, pianos, etc.) The terminology, along with the resonant frequencies, are summarized in the table below. Note that, since the *fundamental* frequency is the *smallest frequency* of standing waves that can be produced on a given string, it must have a harmonic number of 1:  $f_1 =$

$v/2L$ . Therefore, the  $n^{\text{th}}$  harmonic frequency is given by:

$$\text{Strings: } f_n = nv/2L = n f_1, \quad \text{for } n = 1, 2, 3, \dots$$

<b>n</b>	<b>Frequency</b>	<b>Relation</b>	<b>Terminology</b>
1	$f_1 = v/2L$	—	fundamental frequency, or first harmonic
2	$f_2 = 2v/2L$	$f_2 = 2 f_1$	first overtone or second harmonic
3	$f_3 = 3v/2L$	$f_3 = 3 f_1$	second overtone or third harmonic
<i>etc.</i>	<i>etc.</i>	<i>etc.</i>	<i>You get the idea....</i>

Note that the *overtone*s tell us how many “*tones*” we are over the fundamental

frequency (the first harmonic). The harmonic tells us directly the harmonic number,  $n$  (that is, for example, the *fifth* harmonic means  $n = 5$ ). For standing waves on strings, all of the harmonics are possible ( $n = 1, 2, 3, 4, \dots$ ).

Note that the average human ear hears sounds in the approximate frequency range from **20 Hz** to about **20 kHz = 20,000 Hz**. Sounds with frequencies below this range are called *infrasonic* sounds, while sounds with frequencies above this range are called *ultrasonic* sounds. Rhinoceroses communicate with one another using infrasonic sound waves, while bats navigate using ultrasonic waves.

## Solution to Example 23.2

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A certain violin string is **30 cm** long between its fixed ends and has a mass of **2.0 g**. The string sounds a *concert A* (**440 Hz**) when played without fingering. Where must one put one's finger in order to play a *C* note (**528 Hz**)?

Note that, when a stringed instrument is played, the dominant frequency obtained is the *fundamental* frequency, corresponding to  $n = 1$ . We thus have the following data given in the problem statement:

$$L = 0.30 \text{ m} \quad m = 0.0020 \text{ kg} \quad f_1 = 440 \text{ Hz} \quad f_1' = 528 \text{ Hz}$$

Both of the frequencies given are fundamental frequencies, since they are obtained by playing a string (the *same* string!). So why are they different frequencies? Because, by pressing our finger somewhere along the string's length, we are effectively changing the *length* of the string being played, thereby changing the fundamental frequency (that is, the *note* heard). We have denoted the fundamental frequency obtained *with* fingering with a prime ( ' ).

We certainly know that the fundamental frequency is given by

$$f_1 = v/2L .$$

This tells us the *speed* of waves on the string:

$$v = 2L f_1 = 264 \text{ m/s} .$$

We can then get an expression for the fundamental frequency obtained with fingering, since pressing one's finger to the string does not change the string tension or the linear mass density, so that it does not change the wave speed:

$$f_1' = v/2L' .$$

It then follows that the new effective length of the string with fingering must be

$$L' = v/2f_1' = 0.25 \text{ m} .$$

Since the length of the string without fingering is **30 cm**, and that with fingering is **25 cm**, it follows that, to play a *C* note, we must place our finger **30 cm - 25 cm = 5 cm** from the end of the string.

**Summary of important points in this example:**

1. Playing a string ordinarily produces the *fundamental* frequency on the string,  $f_1$ .
2. Changing the length of a string on a stringed instrument by pressing your finger against the string does not change the string's linear mass density or its tension, and therefore does not change the wave speed on the string!

## Standing Waves in Pipes

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For standing waves in strings, we saw that both ends of the string had to be *nodes*. This was because the ends of the string were fixed – they could not move – so they *had* to be nodes (since nodes don't move; antinodes are positions of maximum displacement). If the end of a pipe is *closed*, then it also must correspond to a *node*. However, with pipes it is also possible to have an *open* end. At open ends we must have *antinodes* for standing waves. (This is because the open end of a pipe is an end at which the air readily moves back and forth in a longitudinal pressure/sound wave.) Thus, the conditions for standing waves in a pipe depend on the physical characteristics of the pipe under consideration. There are two types of pipes with which we must be familiar.

### Open Pipes

An *open pipe* is *open* at *both* ends. This means that there must be *antinodes* at both ends (a maximum of air displacement from equilibrium). The equation giving the possible resonant frequencies for standing waves in open pipes is

Open pipes: $f_n = \frac{nv}{2L}$ for $n = 1, 2, 3, \dots$
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where  $L$  is the length of the pipe and  $v$  is the speed of sound in whatever fills the pipe (usually air). A *flute* is an example of an open pipe.

The following table summarizes the frequencies and terminology associated with these frequencies for standing waves in open pipes. (The table is the same as for the standing waves in strings given previously, since the equation for the resonant frequencies is the same.)

<b>n</b>	<b>Frequency</b>	<b>Relation</b>	<b>Terminology</b>
1	$f_1 = v/2L$	—	fundamental frequency, or first harmonic
2	$f_2 = 2v/2L$	$f_2 = 2 f_1$	first overtone or second harmonic
3	$f_3 = 3v/2L$	$f_3 = 3 f_1$	second overtone or third harmonic
<i>etc.</i>	<i>etc.</i>	<i>etc.</i>	<i>etc.</i>

### Closed Pipes

A *closed pipe* is open at *one* end (so the sound can get out!) and *closed* at the other end. Thus, we must have an *antinode* at the first end, and a *node* at the other end. In each of the situations discussed thus far (strings, open pipes), we've had the *same* condition at *both* ends. We now have *different* conditions at the two ends, so it should not surprise us to find out that the equation describing the resonant frequencies is somewhat different:

<b>Closed pipes:</b> $f_n = \frac{nv}{4L}$ for $n = 1, 3, 5, \dots$
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where again the pipe length is denoted  $L$ , and  $v$  is the speed of sound in the medium inside the pipe. Note that in the denominator we now have  $4L$  instead of  $2L$ . Also, we now have only *odd* harmonics. (This is all a result of the geometry associated with the different end conditions brought on by the closed and open ends of these pipes!) An open *coke bottle* is an example of a closed pipe. (Haven't you ever blown across the top of an open coke bottle and heard the various tones as the level of coke left in the bottle is changed?)

The following table summarizes the frequencies and terminology. *Watch out for the overtones!*

<b>n</b>	<b>Frequency</b>	<b>Relation</b>	<b>Terminology</b>
1	$f_1 = v/4L$	—	fundamental frequency, or first harmonic
3	$f_3 = 3v/4L$	$f_3 = 3 f_1$	first overtone or third harmonic
5	$f_5 = 5v/4L$	$f_5 = 5 f_1$	second overtone or fifth harmonic
<i>etc.</i>	<i>etc.</i>	<i>etc.</i>	<i>etc.</i>

Make sure that you understand the harmonic numbers and the corresponding overtones for the strings, open pipes and closed pipes!

## Solution to Example 23.3

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An open organ pipe has a fundamental frequency of **300 Hz** in air. The first overtone produced in a closed organ pipe has the same frequency as the first overtone of the open pipe. How long is each pipe? Use  $v_{air} = 340 \text{ m/s}$ .

This is really quite simple if we are just careful with the terminology. We will use an “(o)” superscript for the *open* pipe, and a “(c)” superscript for the *closed* pipe. Recall that, for a closed pipe, only the *odd* harmonics are present ( $n = 1, 3, 5, \dots$ ). It then follows that the *first* overtone produced corresponds to  $n = 3$ . (The first tone produced over the fundamental.) On the other hand, all harmonics are present for the open pipe ( $n = 1, 2, 3, \dots$ ), so the first overtone produced in it corresponds to  $n = 2$ . We thus have that

$$f_1^{(o)} = 300 \text{ Hz} \quad f_3^{(c)} = f_2^{(o)} \quad \text{and} \quad v = 340 \text{ m/s} .$$

From the given frequency for the open pipe, we immediately get that

$$f_1^{(o)} = v/2L^{(o)}$$

from which

$$L^{(o)} = v/2f_1^{(o)} = \mathbf{57 \text{ cm}} .$$

We also immediately get that

$$f_2^{(o)} = 2 f_1^{(o)} = 600 \text{ Hz} = f_3^{(c)} .$$

But

$$f_3^{(c)} = 3v/4L^{(c)}$$

so that

$$L^{(c)} = 3v/4f_3^{(c)} = \mathbf{43 \text{ cm}} .$$

Note that the whole key to this problem was really just terminology – the jargon associated with music!

## Sample Quiz 23

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A wave is produced on a string which satisfies the relation  $y(x,t) = 0.12 \sin(3x + 5t)$ , where all quantities have *MKS* units.

1. What is the amplitude of the wave?

- a. 0.012 cm    b. 0.12 cm    c. 1.2 cm    d. 12 cm    e. 120 cm

2. What is the wavelength of the wave?

- a. 0.25 m    b. 2.1 m    c. 5.3 m    d. 6.1 m    e. 6.8 m

3. What is the period of the wave?

- a. 0.22 s    b. 0.69 s    c. 1.3 s    d. 2.5 s    e. 4.7 s

4. What is the speed of the wave?

- a. 0.58 m/s    b. 0.81 m/s    c. 0.95 m/s    d. 1.7 m/s    e. 2.4 m/s

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A standing wave is produced on a string whose mass is **2.5 g** and whose length is **1.3 m**. The speed of waves on the string is **320 m/s**.

5. What is the linear mass density of the string?

- a.  $0.89 \times 10^{-3} \text{ kg/m}$     b.  $1.7 \times 10^{-3} \text{ kg/m}$     c.  $1.9 \times 10^{-3} \text{ kg/m}$     d.  $2.1 \times 10^{-3} \text{ kg/m}$     e.  $2.9 \times 10^{-3} \text{ kg/m}$

6. What is the tension in the string?

- a. 25 N    b. 86 N    c. 120 N    d. 175 N    e. 190 N

The standing wave is seen to have two “bumps” or *antinodes* between the two ends of the string.

7. What is the wavelength of the wave?

- a. 0.35 m    b. 0.65 m    c. 1.3 m    d. 2.6 m    e. 3.5 m

8. What is the frequency of the wave?

- a. 250 Hz    b. 340 Hz    c. 120 Hz    d. 75 Hz    e. 550 Hz

## Answers to Sample Quiz 23

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1. d *Watch your units!*
2. b *What is the wavenumber equal to?*
3. c
4. d
5. c
6. e
7. c *How many bumps form one wavelength?*
8. a

## Homework 23

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### I. Warm-up Exercises

1. A traveling wave has a wave speed of **22 m/s** and a wavelength of **0.83 m**. What is the frequency of the wave?
2. A string has a length of **2.5 m** and a mass of **23 g**. What is the linear mass density of the string?
3. What is the wavelength of a wave whose wave number is **3.5 rad/m**?
4. A stretched string has a length of **1.3 m**. The string has a standing wave on it which consists of **3 bumps** between the two ends of the string. What is the wavelength of the wave?
5. A string has a linear mass density of **0.035 kg/m**. Waves traveling on this string have a speed of **14 m/s**. What is the tension in the string?
6. What is the speed of sound in air on a hot summer day when the temperature is **32 °C**?
7. The speed of sound in air at a certain location is **340 m/s**. A closed pipe has a length of **1.3 m**. What is the frequency of the second overtone produced in the pipe?

### II. Some Standards

8. A string is positioned along the x-axis. A wave is imposed on the string which has its motion described by the equation (in MKS units)  $y(x,t) = 0.0015 \sin(kx - \omega t)$ . The wave is seen to have a wavelength of **1.25 m**, and it is found that it takes **0.050 s** for one wavelength to pass by any given point. *Express all of your answers in MKS units!* (a) What is the wave number of the wave? (b) What is its angular frequency? (c) What is the speed of the wave on the string? (d) What is the linear mass density of the string if the tension is **1.5 N**? (e) What is the displacement of the point on the string at  $x = 0.30 \text{ m}$  and at  $t = 0.020 \text{ s}$ ?
9. A horizontal string of mass **0.0035 kg** and total length **1.1 m** is attached to a wall at one end. The other end runs over a pulley and then runs vertically down to a **3.5-kg** mass which hangs from its free end. The length of string between the wall and the pulley is **0.76 m**. (a) What is the tension in the string? (b) What is the speed of waves on the string? (c) What are the lowest

three frequencies that will produce standing waves on the stretched string between the wall and the pulley? (d) What are the wavelengths on the string corresponding to the three frequencies in the previous question?

10. A closed pipe contains air at  $0\text{ }^{\circ}\text{C}$ . The fundamental frequency of sound produced in the pipe is **102 Hz**. (a) What is the length of the pipe? (b) What is the wavelength of the sound at the fundamental frequency? (c) What is the wavelength of the first overtone produced in the pipe?

### III. So, you think you're pretty good...?

11. When two pipes are physically close to one another, vibrations in one pipe can cause pressure waves to enter the second pipe and possibly cause resonant vibrations in the second pipe if the conditions are right. Such induced vibrations are called *sympathetic vibrations*. A closed pipe of length **0.95 m** stands next to an open pipe. Both pipes are in a room in which the temperature is **25  $^{\circ}\text{C}$** . What length of the open pipe will cause sympathetic vibrations in the second overtone when the closed pipe is played in its fundamental mode?

12. Two pulleys are positioned **0.80 m** apart in a room at **22  $^{\circ}\text{C}$** . A string having a linear mass density of **0.0031 kg/m** runs over one pulley and horizontally to the next. The string has a **2.5-kg** mass hanging vertically from each of its ends which hang over each pulley. What *wavelength of sound* is produced if a standing wave is maintained on the string in its second overtone mode of oscillation? (*Hint: The frequency of the wave on the string is the same as the frequency of sound waves produced in the air. This is a general feature of waves – the speed and wavelength can change as the wave goes from one medium to another, but the frequency remains the same! This applies to mechanical waves as well as light waves.*)

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Answers

## Answers to Homework 23

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1. 27 Hz
2. 0.0092 kg/m
3. 1.8 m
4. 0.87 m
5. 6.9 N
6. 350 m/s
7. 327 Hz
8. (a) 5.0 rad/m (b) 130 rad/s (c) 26 m/s (d) 0.0022 kg/m (e)  $-0.0013$  m
9. (a) 34 N (b) 100 m/s (c) 66 Hz ; 130 Hz ; 200 Hz (d) 1.5 m ; 0.77 m ; 0.50 m
10. (a) 0.811 m (b) 3.25 m (c) 1.08 m
11. 5.7 m
12. 2.1 m