L2: Thin Lenses

In the last lecture we discussed how light *bends*, or *refracts*, when it goes from one medium to another. In this lecture we see how to exploit this refraction of light to cause parallel light rays to focus either to or from a point. These points are called *focal points*, and the object doing the refracting is called a *lens*. In our discussion, we will assume that the distance from one side of the lens to the other is much smaller than the focal length of the lens (the distance from the center of the lens to the focal point). Lenses satisfying this approximation are called *thin lenses*.

Diverging Rays

We see an object by viewing the light either emitted or reflected (or both) by the object. Once the light has left the object, the light rays spread apart as they move further and further away. Such light rays are said to *diverge*. (Light rays that come together as they move are said to *converge*).

Light leaving any real object always diverges.

The pupils in our eyes are perhaps a few millimeters in diameter. If we view an object which is close to us, the light rays from the object diverge as they move through the pupil and into the eye, where they are made to converge onto the retina by the lensing action of the eye. As we move farther and farther away from the object, however, the light rays that are intercepted by our eyes' pupils tend to be more and more parallel, since the light rays reaching our eyes are such a small fraction of the light emitted in all directions by the object. For objects that are *very* far away (the sun is a good example), we may be confident in assuming that the light rays are essentially parallel. (The lights rays are *actually* still diverging – only light from an infinitely far object actually *is* parallel – but the divergence of the light is so slight that we could hardly detect it even if we wanted to....)

In the future, whenever we speak of *parallel light rays*, you may think of the light as if it had come from a very distant object.

Types of Thin Lenses

We usually think of a thin lens as a thin, circular piece of clear material such as glass or plastic. The lens is usually in air. Light travels from one side of the lens, through the lens where it is refracted at the first and second interfaces between the air and the material making up the lens, and back out into the air on the other side of the lens. Light can therefore be on either side of the lens.

There are two *focal points* associated with the two sides of the lens on which the light can be found. The *first focal point*, F_1, is that point from which light can diverge (such as light coming from a very small light bulb placed at that position) such that it will pass through the lens and emerge as *parallel light*. The *second focal point*, F_2, is the point to which parallel light is *focused* (converges to a point) by the lens. The two focal points, F_1 and F_2, are on opposite sides of the lens. The ***focal distance*** F is the distance from the center of the lens to either focal point (they're both the same distance from the lens' center).

A ***converging lens*** is a lens which takes parallel light rays and *converges* that light to a point – the focal point, F_2. The point F_2 is thus on the side of the lens which is opposite the side of the incident light, and F_1 is on the same side as the incident light. (See the figure below.) The ***focal length, f,*** of a *converging* lens is equal to *positive* the focal distance:

focal length = + focal distance $f = +F$ *Converging Lens*

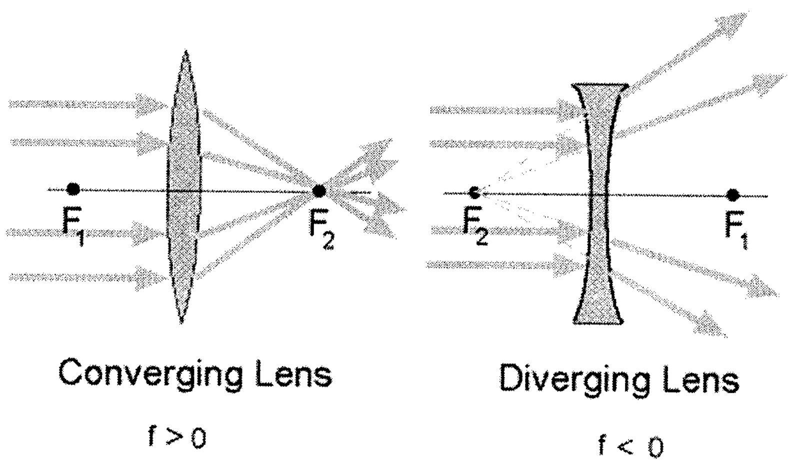


Figure 2.1

On the other hand, a ***diverging lens*** is a lens which takes parallel light and makes the light appear to *diverge* from the point F_2 as it leaves the lens, so that F_2 must be on the same side of the diverging lens as the incident light, and F_1 is on the opposite side. (Things are kind-of opposite for the diverging lens as for the converging lens.) The *focal length* for the *diverging* lens is taken to be *negative* the focal distance:

focal length = negative the focal distance $f = -F$ *Diverging Lens*

When a small object is at the point F_1 for a *converging* lens, the diverging light which approaches the lens from the object is turned into parallel light by the refraction of the lens. For a *diverging* lens, light which approaches the lens converging towards the point F_1 (on the other side of the lens) emerges from the lens as parallel rays of light. See the figure below.

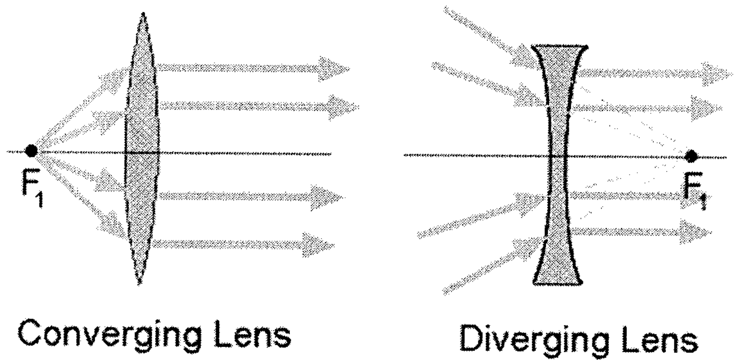


Figure 2.2

Lens Images

We now consider the case in which we place a real object at some position in front of a thin lens. By a *real object* we mean something that we can hold in our hands and put in front of a lens or mirror. Things like a marble, an ant, a thumb tack, a small light bulb, or your irritating neighbor are all examples of real objects. The distance from the center of the lens to the real object is called the *object distance*, and is denoted D_o. The distance from the center of the lens to the location of the image of the object formed by the lens is called the *image distance*, D_i. *These distances are always positive.* What may be positive or negative are what we will call the *object position* and the *image position*, denoted d_o and d_i, respectively.

The positions of real objects and real images are always positive.

The positions of virtual objects and virtual images are always negative.

We shall discuss virtual objects and images later on in this lecture. Note the convention that we are setting up, however: **Capital letters represent real *distances* which can be measured by a meter stick, and are therefore always positive (F, D_o, D_i).** Small letters represent *positions* (f, d_o, d_i); we use the *sign* of the position to give us some information about the location of the point under consideration relative to the lens as determined by the characteristics of the light (diverging, converging) as it approaches and emerges from the lens (this will be clarified a bit later in this lecture).

Note: The terminology and notation that we are introducing here is not standard if you look in other references. It is being introduced in an attempt to help clarify the confusing sign convention that *is* standard for this material!

Locating Images

There are two ways to approach locating and describing the image of an object in a thin lens. The first method is the *analytical method* which uses mathematics to determine the image location, size, orientation, *etc*. The equation used to help us accomplish this feat is appropriately called the ***thin lens equation***:

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}\ . \qquad (2.1)$$

This equation relates the focal length to the object and image positions. If we know any two of these quantities, we can determine the third using this equation.

The second way to find and characterize an image is to use a ***ray diagram***, which exploits the geometry associated with thin lenses.

> *It is assumed when applying the rules for drawing ray diagrams that the rays being drawn are all paraxial rays, which means that the rays are nearly parallel to one another. This means in more pedestrian terms that the object height should be small compared to the object distance. The larger the object height is compared to its distance from the lens, the worse will be the results from the ray diagram.*

Before going into any more details associated with locating and describing an image in a thin lens (and there will be *plenty* of more details coming up for those of you who just *love* details!), let's first do an example showing how to use the thin lens equation.

Example 2.1

A small sphere of height **1.3 cm** is placed **20 cm** in front of a lens of focal distance **10 cm**. Find the value of the image position d_i if (a) the lens is a converging lens. (b) the lens is a diverging lens.

Answers: (a) + 20 cm (b) –6.7 cm

Solution

Solution to Example 2.1

A small sphere of height **1.3 cm** is placed **20 cm** in front of a lens of focal distance **10 cm**. Find the value of the image position d_i if (a) the lens is a converging lens. (b) the lens is a diverging lens.

The small sphere is the object; we call its height h_o: $h_o = 1.3$ cm. The object is placed *20 cm* from the lens, so the *object distance* is $D_o = 20$ *cm*. Since the object is a *real* object (we *really* put something there!) the *object position* must be *positive*. Thus, $d_o = +D_o = +20$ *cm*. The *focal distance* of the lens is $F = 10$ *cm*.

(a) Since the lens is a *converging* lens the *focal length* must be *positive*, so that $f = +F = +10$ *cm*. From the *thin lens equation* we thus get that

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

or

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$$

so that $d_i = +20$ *cm*. Note that the *positive* image position tells us that this image must be *real*.

(b) Since the lens is a *diverging* lens the focal length must be *negative*, so that $f = -F = -10$ *cm*. Therefore,

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-10} - \frac{1}{20} = -\frac{3}{20}$$

so that $d_i = -20/3$ *cm* $= -6.7$ *cm*. Note that the *negative* image position tells us that this image must be *virtual*.

More with Images

As discussed earlier, light *diverges* as it leaves any real object. We shall use this as the definition of a *real object* – one for which the light *diverges* from the object's position, called the object point, *O*, as it approaches the thin lens. If the light approaching a thin lens *converges* toward some point (the *object point*, O) on the other side of the lens, then the light cannot possibly be coming directly from a real object. In this case, we say that the object for the lens is a *virtual object*. (In such a case, the light must be coming from another lens or a mirror which has taken the diverging light from a real object and is making it converge toward the thin lens under consideration. We are not concerned at this point with *how* the light approaching the lens came to be converging. Rather, we are simply acknowledging the fact that it *is* converging as it approaches the lens by saying that the object distance–the distance from the center of the lens to the point to which the light is converging–is that for a *virtual* object.)

Now consider the light as it leaves the other side of the lens. This light will either converge or diverge as it leaves the lens.

If the light is <u>converging to</u> some point as it leaves the lens, then that point, called the *image point, I*, is *real* and the corresponding image position is *positive*.

Thus, if the distance from the lens to the image point is D_i, then it follows that, for a **real** image, the image position d_i is given by: $\mathbf{d_i = +D_i}$.

Light that is focused to a real image point can be projected onto a screen or a piece of paper placed at the image point. (Hence the term "real" image. The light will, of course, diverge once it moves beyond this real image point, just as if it were leaving a real object....)

On the other hand, if the light as it leaves the lens tends to *diverge*, then it must be diverging from some point, called the *image point, I*, on the *other* side of the lens. Light after leaving the lens does not form a real image–that is, it does not focus to a point. The image point in this case is therefore said to be a *virtual point*, and the corresponding image position is *negative*.

If the light is <u>diverging from</u> some point as it leaves the lens, then that point, called the *image point, I*, is *virtual* and the corresponding image position is *negative*.

Thus, if the distance from the lens to the image point is D_i, then it follows that, for a **virtual** image, the image position d_i is given by: $\mathbf{d_i = -D_i}$.

Light from a virtual image point will not form a clear picture on a screen or piece of paper, but it can be focussed by our eye if we look directly into the light as it leaves the virtual image point.

It sometimes confuses people to hear that our eyes cannot focus light that is converging toward a *real* image point, but we can readily focus on light that is diverging from a *virtual* image point. If you think about it for a minute the reason should be clear: Our eyes are configured to view *real objects*. Light diverges as it leaves real objects, which means that the optics of our eyes is made to converge the diverging light entering our pupils onto the retinas in the back of our eyes. Light coming from a *virtual image point* is also diverging. This means that our eyes can readily focus it onto the retina so we can see the image.

However, light heading toward a *real image point* is *converging*; our eyes cannot focus light onto the retina that is already converging before it enters the optical system of our eye. For example, your image in a flat (plane) mirror is a virtual image – you have no trouble viewing it. But have you ever stood directly in front of a screen onto which the image of a slide was being projected? If you stand back and look at the image once it has been projected onto the screen, you can readily view it. However, if you stand in front of the screen and look at the light coming from the projector before it reaches the screen, you cannot make out the picture that is being projected. You cannot make out a real image from the light as it approaches the image point (the image point on the screen in the case of the projector).

Now that we have defined the object and image positions, d_o and d_i, we can define the ***magnification, m*** of the image. The *magnitude* of the magnification, $|m|$, gives us exactly what we would expect it to give us: it tells us the ratio of the image height to the object height. (How much bigger is the image than the object? ...by how much is it *magnified*?)

$$|m| = \frac{h_i}{h_o}. \qquad (2.2)$$

The *sign* of the magnification tells us whether the image is right-side up or upside down relative to the object – that is, it tells us whether the image is *upright* or *inverted* relative to the object:

If $m > 0$, then the image is *upright*.

If $m < 0$, then the image is *inverted*.

The magnification for a given thin lens can be computed from the object and image positions as follows:

$$m = -\frac{d_i}{d_o}. \qquad (2.3)$$

Note that you must be very careful to use the appropriate *signs* for the image and object positions in Eq. (2.3) above!

Solution to Example 2.2

Locate and describe the images for parts (a) and (b) in Ex. 2.1.

(a) We got in Ex. 2.1 that $d_i = +\ 20\ cm$. The fact that the image position is *positive* means that the image must be a *real* image. This means that light rays leaving the lens must *converge to a point* (the *image point*), so the image must be *20 cm* to the *right* of the lens (assuming that the light enters the lens from the left). The *magnification* of the object by the lens is

$$m = -\frac{d_i}{d_o} = -\frac{20\,\text{cm}}{20\,\text{cm}} = -1.$$

The fact that the magnification is *negative* tells us that the image is *inverted* relative to the object. The *magnitude* of the magnification, $|m| = 1$, tells us that the image is the same size as the object:

$$|m| = 1 = \frac{h_i}{h_o}$$

so that the image height is $h_i = |m|\ h_o = h_o = 1.3\ cm$.

(b) In this case we got previously that $d_i = -6.7\ cm$. The negative sign means that the image is a *virtual* image. This tells us that light does not *converge to* the image point, which means that it must *divergeaway from* the image point, *as if* it had come from that point (even thought it didn't; we'll see more clearly what this means when we do ray diagrams in the next section). This means that the image point must be *6.7 cm* to the *left* of the lens. The magnification of the object in the lens is

$$m = -\frac{d_i}{d_o} = -\frac{-6.7\,\text{cm}}{20\,\text{cm}} = +0.34.$$

The positive sign tells us that the image has the same orientation as the object (so if the object is upside-down, then so is the image), and the image is *0.34* times as big as the object. In particular,

$$|m| = 0.34 = \frac{h_i}{h_o}$$

from which $h_i = 0.34\ h_o = 0.44\ cm$.

Ray Diagrams

We have already seen how to locate and describe the image formed by a thin lens using the *thin lens equation.* We will now see how to accomplish the same feat by using *ray diagrams*. In an attempt to keep the ray diagram as uncluttered as possible, we shall draw a *vertical line* to represent the thin lens. If the lens is a *converging* lens, then we will place a (+) above the lens, denoting the fact that the focal length is positive for a converging lens. If the lens is a *diverging* lens, then we will place a (–) above the vertical line representing the lens. Furthermore, we shall use an *arrow* to represent the object and corresponding image. We can do this since there are really only three things that we need to know about the object and image: their orientation (up or down), their size, and their location, all of which can be represented by the arrow.

We start the construction of a ray diagram by drawing the vertical line representing the lens. We then draw a horizontal line through the center of the lens. This horizontal line represents the *optical axis*. A scale must then be determined (for example, *1 cm* in the diagram corresponds to *10 cm* in the real system being studied). Once the scale has been specified, the focal points F_1 and F_2 can be drawn and labeled. (We will always assume that light is incident from the left in the ray diagrams we draw in these lecture notes. You may draw the light coming from whichever direction you wish.) The diagram below shows this set-up for the case of an object placed *15 cm* in front of a *10-cm focal length* converging lens.

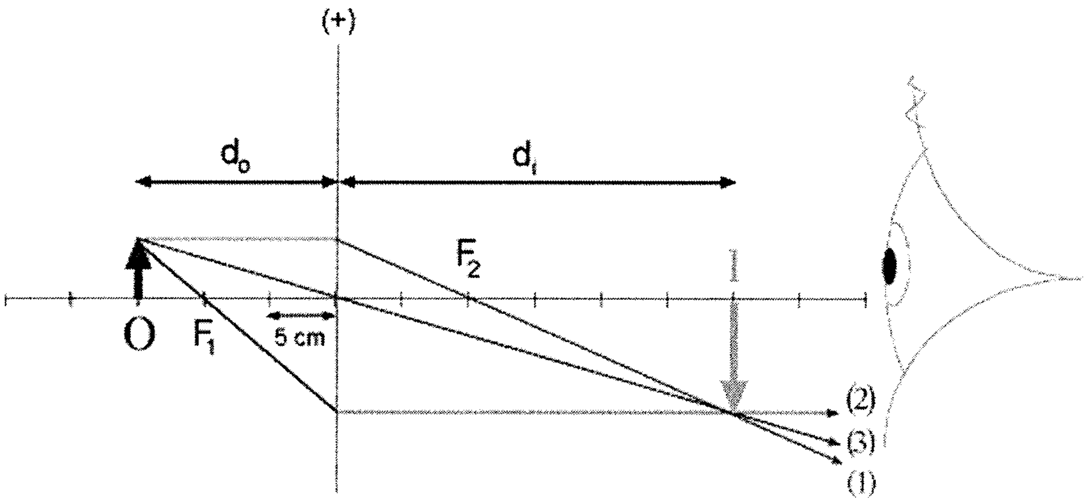


Figure 2.3

When drawing a ray diagram, it is always important to remember that light is reflected or emitted by every point on the object. This light, once emitted, diverges in all directions. The bottom of the arrow representing the object lies on the optical axis. The image of this point will always also lie on the optical axis. Therefore, all we need do is determine the location of the image of the *tip* of the arrow representing the object. Once we locate its image, we can immediately draw in the full arrow representing the image of the object in the lens. (See Fig. 2.3. The next example will also demonstrate this.)

It only take two rays to locate an image point – it is the point where the two rays intersect. We shall learn how to draw the paths of *three* rays leaving the tip of the arrow of the object (out of the *infinite* number of rays leaving that arrow tip!) and passing through the lens. The rules for drawing these three rays are stated below. See if you can follow the rules stated below on the ray diagram in Fig. 2.3 above (the rays are labeled with the rule numbers given below).

Rules for Drawing Rays

> Ray 1: *The ray leaving the tip of the object traveling parallel to the optical axis will pass through the focal point* F_2 *after passing through the lens.*

> Ray 2: *The ray leaving the tip of the object and passing through the focal point* F_1 *will emerge from the lens traveling parallel to the optical axis.*

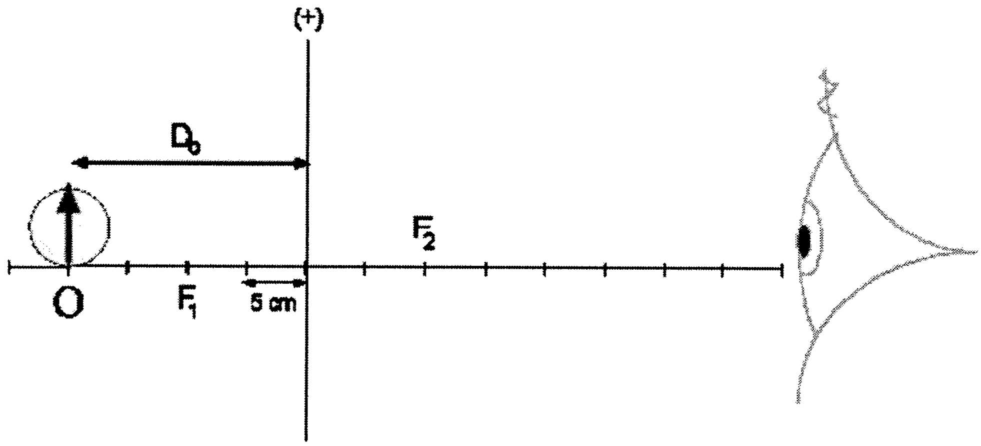
> Ray 3: *The ray leaving the tip of the object and passing through the center of the lens will emerge from the lens undeflected.*

The following examples will refer to these rules by number when drawing the ray diagrams in the example solutions.

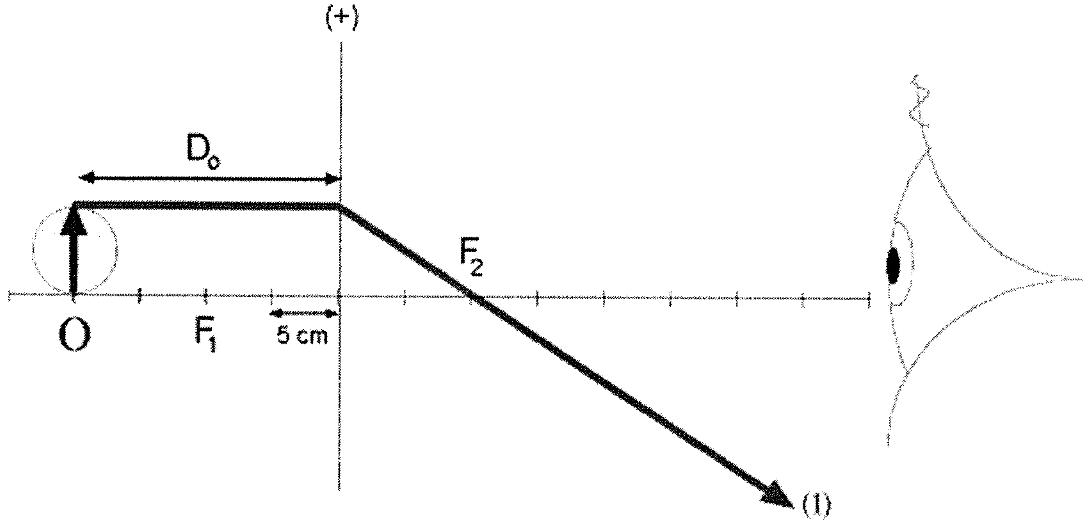
Solution to Example 2.3

Solve Examples 2.1 and 2.2 using ray diagrams.

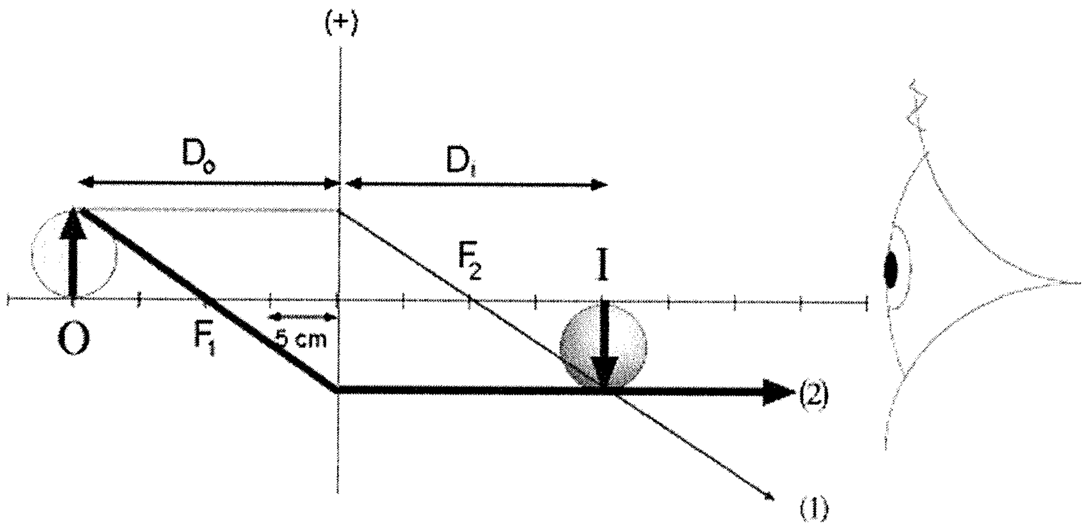
(a) We start by drawing a horizontal line representing the *optical axis*. We then draw in the position of the lens with a vertical line, and put a (+) above it to denote the fact that it is a *converging* lens (with a *positive* focal length). For light coming in from the left (as is our convention) the first focal point F_1 is *10 cm* to the *left* of the lens, and the second focal point F_2 is *10 cm* to the *right* of the lens. We make up a scale and draw tick-marks, and then draw in the points F_1 and F_2 at the appropriate positions. We can then draw in an arrow representing the object at its appropriate location, in this case *20 cm* to the left of the lens. This completes the general set-up for this problem. This set-up is shown in the figure below.



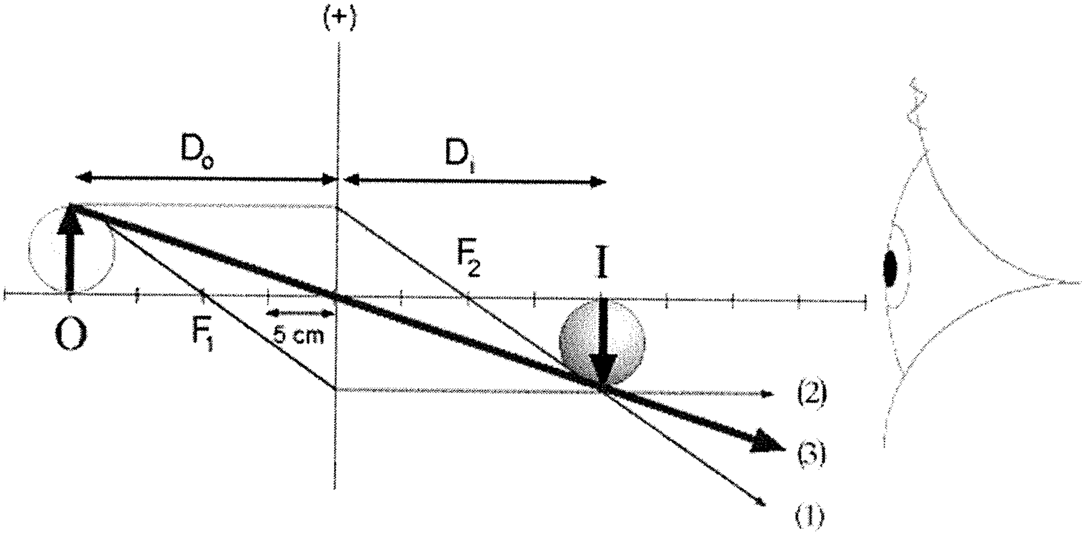
We can then draw in *Ray 1*, which starts at the tip of the arrow representing the object, passes *parallel to the axis*, and then emerges from the lens and passes straight through the focal point F_2. This set-up with Ray 1 drawn in is shown in the figure below.



We can then draw in *Ray 2*, which starts at the tip of the arrow representing the object and passes straight through the focal point F_1 on its way to the lens. It then emerges from the lens moving *parallel* to the optical axis. Once the first two rays have been drawn the image location can be determined: it is the point where the two rays intersect. The image can then be drawn as an arrow which starts on the optical axis and whose tip is at the intersection point of the two rays (this is the image of the tip of the object). Ray 2 long with the image is shown in the figure below.



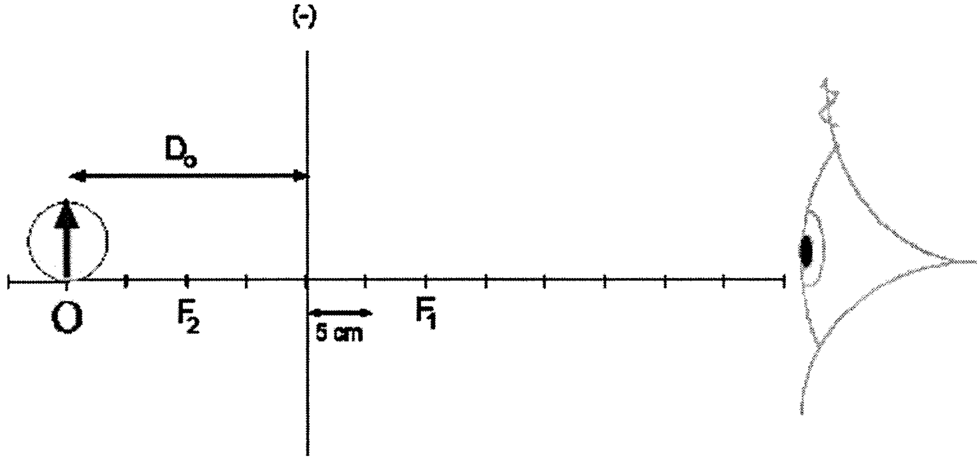
We now have our answer: we can simply read off of the diagram the location of the image, and we can inspect the diagram for the relative orientation of the image (relative to the object) and its approximate size. (The larger you make your ray diagram and the more carefully you use your ruler, the more accurate will be your results.) We can see that the image is about *20 cm* to the right of the lens, it is *inverted* relative to the object, and it is about the same size. However, it is *always* a good idea to try to draw in the third ray to verify that you have done everything correctly so far—if *Ray 3* does not pass through the image point (to within uncertainties) then you have done something wrong. *Ray 3* starts at the tip of the object and passes undeflected straight through the center of the lens (that is, where the vertical line representing the lens crosses the optical axis). This ray is shown in the figure below.



It can be seen that the three rays do indeed pass through the same point. (If you are sketching these rays diagram by hand without a ruler your results will, of course, be only approximate.) The ray diagrams therefore verifies our previous results for the converging lens.

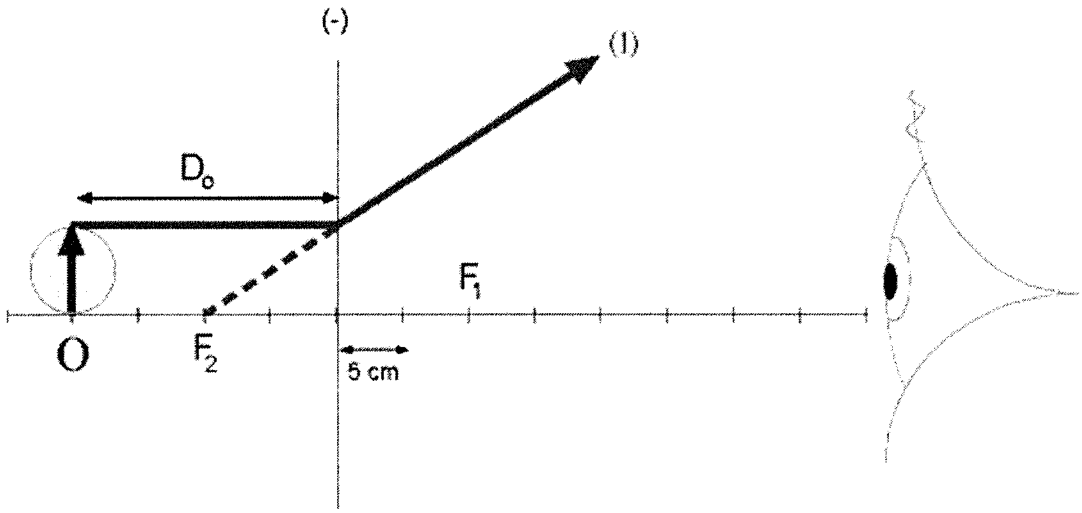
(b) We must now deal with the case of a *diverging* lens. This case will be a bit trickier than the converging lens was in the previous part. (Although the case of a converging lens is not always so straight-forward, as your homework will show you!) Nevertheless, this is still rather straight-forward once you get the idea of how they work. Let's take it a step at a time.

As before, we start by drawing the optical axis, a vertical line with a (–) above it to represent the *diverging* lens (with a *negative* focal length), defining a scale, drawing tick marks, labeling the point F_1, now to the *right* of the lens (since the lens is *diverging* the focal point positions are switched!), and F_2 to the *left* of the lens. The arrow representing the object can then be drawn in at the appropriate location. (*See the figure below for the set-up so far.*)



To draw *Ray 1*, we must first think a bit. *Ray 1* leaves the tip of the object and travels parallel to the axis toward the lens. It then leaves the lens and passes through the focal point F_2. *But if it leaves the lens moving toward the right, it can't pass through F_2 since F_2 is to the left of the lens!* What are we to do?

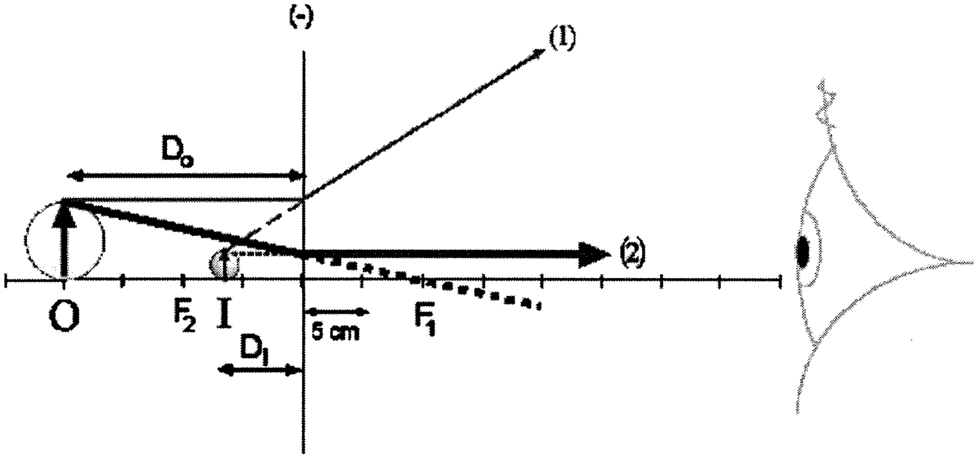
When faced with this situation in optics, in which a ray is supposed to pass through a point which it cannot physically pass through, we simply extend the ray (forwards or backwards, whichever works) so that it *does* pass through the desired point. In this case, *Ray 1* cannot pass through F_2 after it passes through the lens, but it can *appear* as if it has *already* passed through F_2 if we extend the ray *backwards*. This is shown by the dashed line in the figure below.



Again, *Ray 1* did not pass through F_2, but it emerges from the lens looking as though it is coming from F2.

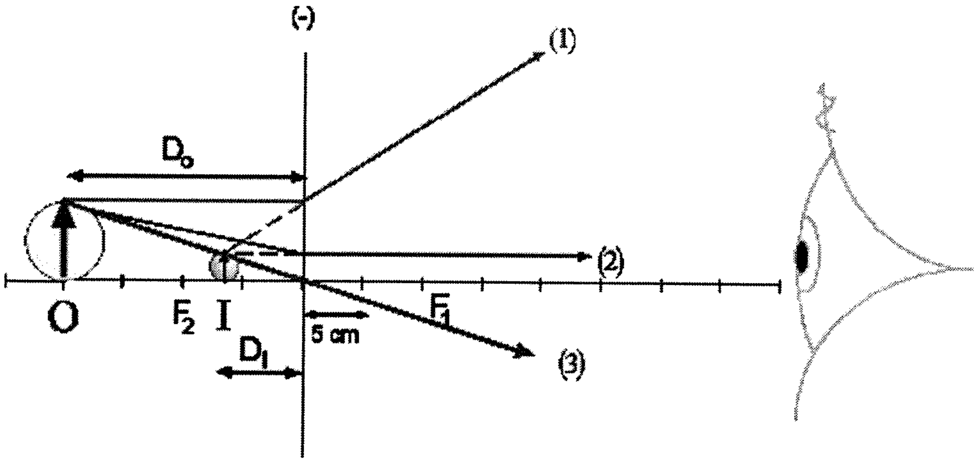
We now draw *Ray 2*. This ray is to leave the tip of the object arrow and pass through the point F_1 on its way to the lens. This ray will then emerge parallel to the optical axis.

The problem, again, is that the ray *cannot* pass through F_1 on its way to the lens since F_1 is on the *other side* of the lens—the lens gets in the way! The way around this is that, instead of having the ray pass through F_1 on its way to the lens, we just have it *head for* the point F_1 as it approaches the lens so that, if the ray is extended forwards beyond the lens, it *will* pass through F_1 (even though it never gets there since the lens gets in the way!). Once this ray reaches the lens, it emerges parallel to the axis. this ray is shown in the following figure.



Again, it only takes *two* rays to determine the image point. the image point is the point where the two rays emerging from the lens intersect one another. Our next problem is that the two rays emerging from the lens do not converge at all—instead they *diverge* from one another! This is the characteristic of a *virtual image*. The light rays do not converge to a point (where they intersect one another) as with a *real image*. Instead, the light rays for a virtual image seem to diverge from a point (even though they really didn't come from that point). To see where this point is, simply trace the rays emerging from the right-hand side of the lens *backwards* to the other side of the lens. Where these two backwards-drawn rays intersect is the (virtual) image point. This point (along with the dashed rays drawn backwards) is shown in the figure above and is labeled *I*. We can see that this image is *virtual* (since the light rays didn't actually converge at that point after leaving the lens), it is *upright* (that is, it has the same orientation as the object), it is *minified* (that is, it is *smaller* than the object), and it is about *7 cm* from the lens. To within uncertainties associated with estimating the image distance on our diagram, these results agree with the results obtained earlier in Ex. 2.2.

As with the solution in part *(a)*, it is important to check our results by drawing in the third ray. This ray leaves the tip of the arrow representing the object and passes straight through the center of the lens. This ray is shown in the diagram below.



We can see that this last ray does indeed pass through the image point. (More specifically, since this is a *virtual* image, it might be more appropriate to say that the ray emerging from the lens appears to pass through the image point when traced backwards through the lens; however, since this last ray is undeflected as it passes through the lens, it is the only ray that actually passes through the virtual image point in question.)

Make sure that you completely understand the reasoning in drawing the rays in the two parts of this example. The thinking in part *(b)* is simply that if a ray cannot pass through a desired point when following the rules to draw the ray, then trace the ray *backwards* so that the ray appears to have already passed through the desired point, or extend it forwards to have it head for the desired point, even if it never gets there.

Multiple Lens Systems

When an optical system consists of more than one lens (for example, in a compound microscope, or in a refracting telescope), then the method for finding the final image is very similar to the procedure already outlined for finding the image in a single-lens system. Given an original object, consider first only the first lens, ignoring the rest. Locate the image of the object in the first lens; call this image I_1. Using this image and the object, find the magnification in the first lens, $m_1 = -d_{i1}/d_{o1}$. Then treat the image I_1 as the *object* for the *second* lens, O_2. Ignoring all of the other lenses, find the image of this object in the second lens. (This is where it becomes possible to have a *virtual* object! Read the discussion at the beginning of the section labeled *More with Images*.) Also find the second magnification, $m_2 = -d_{i2}/d_{o2}$. Continue this process until all of the lenses have been taken into account. This first of all locates the final image (given by the final image position from the last lens considered). The *total magnification* of the final image relative to the original object, m_{total}, is then simply given by the *product* of all of the individual magnifications for each lens in the system:

$$m_{total} = m_1 \cdot m_2 \cdot m_3 \cdots \quad (2.4)$$

The interpretation of the magnification of the final image relative to the original object then follows in exactly the same way as for a single lens.

Sample Quiz 2

1. As far as lenses are concerned, an *object* is considered to be a *virtual object* whenever

a. the object distance is greater than the focal length. b. the object distance is less than the focal length. c. the light converges as it approaches the lens. d. the light diverges as it approaches the lens. e. *None of the above.*

2. A *converging lens* is also sometimes called a *positive lens* because

a. $d_o > 0$ b. $d_i > 0$ c. $f > 0$ d. $R > 0$ e. *All of these.*

3. The difference between the *image distance,* D_i, and the *image position,* d_i, is that

a. the distance is always positive, whereas the position can be positive or negative.

b. the position is always positive, whereas the distance can be positive or negative.

c. the distance is measured to the right of the lens, but the position is measured to the left.

d. the distance is measured to the left of the lens, but the position is measured to the right.

e. There is no difference–they are two different terms for the same thing.

4. If the *magnification* of a lens is *negative*, then

a. the image has the same orientation as the object.

b. the image has an inverted orientation relative to the object.

c. the image is real and the object is virtual. d. the image is virtual and the object is real.

e. both the image and object are virtual.

5. There are two good ways to find a description of an image in a lens. One is to use the thin lens equation. The other is to use

a. the lensmaker's equation. b. Snell's law. c. the law of refraction.

d. a free-body diagram. e. a ray diagram.

Answers

Answers to Sample Quiz 2

1. c

2. c

3. a

4. b

5. e

Homework 2

Find and describe the image of the object in the situations described below in problems 1 through 7. Will the image focus on a piece of paper held at the image position? You should be able to solve these problems using both analytical and graphical (ray diagram) methods.

1. A **1.5-cm** tall candle stands **30 cm** in front of a **10-cm** focal distance converging lens.

2. A **1.5-cm** tall candle stands **20 cm** in front of a **10-cm** focal distance converging lens.

3. A **1.5-cm** tall candle stands **15 cm** in front of a **10-cm** focal distance converging lens.

4. A **1.5-cm** tall candle stands **10 cm** in front of a **10-cm** focal distance converging lens.

5. A **1.5-cm** tall candle stands **5.0 cm** in front of a **10-cm** focal distance converging lens.

6. A **1.5-cm** tall candle stands **15 cm** in front of a **10-cm** focal distance diverging lens.

7. A **1.5-cm** tall candle stands **5.0 cm** in front of a **10-cm** focal distance diverging lens.

8. A **3.00-cm** long roach stands facing a **20.0-cm** focal distance diverging lens. The roach's head is **10.0 cm** from the center of the lens. You view the bug through the lens. (a) How far from the lens does the bug's head appear to be as viewed through the lens? (b) How long does the bug seem to be when viewed through the lens?

9. A slide projector is used to project an image of a waterfall from a slide onto a screen which is **7.20 m** from the slide. The slide is **3.70 cm** from the center of the lens. (a) What is the focal length of the lens being used? (b) Could you make out the image of the waterfall if you stood in front of the screen and looked directly into the incoming light from the projector? Why or why not? (c) How much larger than the slide is the image of the slide on the screen? (d) Is the image upright or inverted? (e) Why are slides inserted upside-down into the slide projectors?

The set-up in the following problem is exactly the same as that in the last problem. Let's make sure that you did it correctly....

10. A **3.0-cm** tall object is placed **45 cm** from a screen. A lens is placed between the object and the screen a distance of **15 cm** from the object. A sharp image of the object is seen on the screen. (a) What is the focal length of the lens? (b) Describe the image.

When more than one lens is used in an optical system (such as in a compound microscope or a telescope), the object is simply imaged in the first lens, and then the image of the first lens is used as the object for the second lens. The total magnification is then simply the product of the individual magnifications of the two lenses. With this in mind, try the following problem (it's really not as bad as it looks—things are just broken *down into little steps for you!):*

11. An object of height $\mathbf{h_{01}} = \mathbf{2.0\ cm}$ (*the height of the object for the first lens*) is placed a distance of $\mathbf{d_{01}} = \mathbf{17\ cm}$ in front Lens 1 which is a converging lens and has a focal distance of $\mathbf{f_1} = \mathbf{20\ cm}$. There is a separation of $\mathbf{S} = \mathbf{22\ cm}$ between Lens 1 and a second converging lens, Lens 2, which has focal distance of $\mathbf{f_2} = \mathbf{10\ cm}$. We will approach finding the final image of the object in this two-lens system by following the steps outlined. (a) First, looking only at the first lens and ignoring the second, what is the image position for the object in Lens 1? (b) What is the orientation of this image relative to the object? (c) Is this a real or virtual image? (d) What is the distance between the image and Lens 1? (e) What is the distance between the image (from the first lens) and Lens 2? (f) Does light converge or diverge as it reaches Lens 2? (A rough ray diagram for finding the image in Lens 1, and then showing the position of Lens 2 might be helpful here.) (g) We now treat the image from Lens 1 as the object for Lens 2. Looking only at Lens 2 now, what is the object position for Lens 2? (h) What is the image position for Lens 2? (i) What is the orientation of the image for Lens 2 relative to that of the object for Lens 2? (j) What is the total magnification for this two-lens system? (k) What is the orientation of the final image (for Lens 2) relative to that of the original object (for Lens 1)? (l) What is the height of the final image? (m) Is the final image real or virtual? (n) Would you be able to see the final image if you looked through the double-lens system at the object?

Answers

Answers to Homework 2

Note: Real images can be viewed on a piece of paper placed at the image position; virtual images cannot be viewed on a piece of paper.

1. Image is: 15 cm from lens; real; inverted; 0.75-cm tall

2. Image is: 20 cm from lens; real; inverted; 1.5-cm tall

3. Image is: 30 cm from lens; real; inverted; 3.0-cm tall

4. Image is: infinitely far from lens and is infinitely large (another way of viewing this is to say that no image is formed within a finite distance of the lens)

5. Image is: 10 cm from lens; virtual; upright; 3.0-cm tall

6. Image is: 6.0 cm from lens; virtual; upright; 0.60-cm tall

7. Image is: 3.3 cm from lens; virtual; upright; 1.0-cm tall

8. (a) 6.67 cm (b) 1.21 cm

9. (a) + 3.68 cm (b) No – it's a real image! (c) 194 (d) inverted (e) See the answer to part (d)!

10. (a) + 10 cm (b) 30 cm from lens; real; inverted; 6.0-cm tall

11. (a) –113 cm (b) It is upright (same orientation). (c) virtual (d) 113 cm (e) 135 cm (f) diverges (g) + 135 cm, since it's a real object (h) +10.8 cm (i) inverted (j) –0.53 (k) inverted (l) 1.1 cm (m) real (n) No–your eye cannot focus on converging light; the light coming out of the second lens is converging towards the image point for Lens 2.